Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 170 (2017) 112–144
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

# Efficient ex-ante stabilization of firms

David M. Frankel [a,b,1]

[a] *Melbourne Business School, 200 Leicester Street, Carlton, VIC 3053, Australia*
[b] *Iowa State University, Ames, IA 50011, United States*

Received 3 February 2015; final version received 22 March 2017; accepted 3 April 2017
Available online 19 April 2017

**Abstract**

Distressed firms are vulnerable to inefficient panic-based runs of their workers, suppliers, and customers. A policymaker may try to prevent such a run by pledging to protect the interests of these stakeholders should a firm cease to do business. However, this promise also enables the firm to demand better terms of trade from its stakeholders, which blunts the policy's effectiveness. We show how to avoid such an adverse response by the use of partial, countercyclical insurance. Under certain conditions, such a scheme costlessly implements the first-best outcome in the limit as the stakeholders' information becomes precise. We also identify least-cost efficient schemes in the cases of large noise, learning, and duopoly.


*JEL classification:* C72; D42; D62; G38; H21

*Keywords:* Financial distress; Bankruptcy; Insurance; Global games; Learning; Duopoly

## 1. Introduction

In the 2008/9 crisis, the U.S. government intervened to support or take over several large firms and banks whose failure was regarded as posing a systemic risk.[2] While stockholders were often

*E-mail address:* dfrankel@msn.com.
[1] I thank seminar participants at Bonn, CORE, U. Edinburgh, U. Essex, U. Exeter, Hebrew U., U. Iowa, Melbourne Business School, NYU, Northeastern U., Oxford, Pompeu Fabra, U. St. Andrews, U. Southampton, and University of Technology – Sydney, as well as Alessandro Pavan (the editor) and four anonymous referees.
[2] Examples include Bear Stearns, IndyMac, Fannie Mae, Freddie Mac, AIG, Washington Mutual, General Motors, and Chrysler (Longstaff, 2010, p. 441).

http://dx.doi.org/10.1016/j.jet.2017.04.001
0022-0531/

treated harshly during these bailouts,[3] attempts were made to protect the firms' small stakeholders: its customers, suppliers, and workers. Examples include the G.M. and Chrysler bailouts, which included promises to honor the automakers' warranties and debts to parts suppliers (U.S. Government Accountability Office, 2009). Official explanations of government bailouts often contain language designed to allay the fears of such stakeholders:

> The goals of the conservatorship are to help restore confidence in the Company [Fannie Mae and Freddie Mac] . . . . There is no reason for concern regarding the ongoing operations of the Company. The Company's operation will not be impaired and business will continue without interruption.[4]

Why protect small stakeholders? In a reorganization bankruptcy, their contracts with the firm may be renegotiated. In a liquidation, these contracts will be voided. Hence, sensing that a firm is in distress, some stakeholders may choose not to do business with the firm.[5] This worsens the firm's distress, which gives other stakeholders an even stronger incentive to shun the firm. As the firm cannot function without customers, workers, and suppliers, such a process will lead to liquidation if left unchecked. Liquidation is inefficient since it harms or destroys the value of any investments the stakeholders have made in a relationship with the firm.[6] And in the case of a systemically important firm, it may pose risks to the wider economy.

These concerns create a rationale for instituting policies aimed at reducing the risk of small stakeholder runs. However, if not carefully designed, such policies may cause moral hazard on the firm's part. Intuitively, the added protections make stakeholders less prone to abandon the firm. The firm will be tempted to exploit their greater loyalty by demanding more favorable

[3] Holders of General Motors and Chrysler stock were essentially wiped out ("A Primer on the G.M. Bankruptcy", New York Times, June 1, 2009). Bear Stearns stockholders were paid the equivalent of $10 per share; their stock price had peaked at $172.69 fourteen months earlier (Greenberg, 2010, pp. 4, 184). Voting rights and dividends of Fannie Mae and Freddie Mac stock were suspended ("Fannie Mae and Freddie Mac in Conservatorship," Mark Jickling, Congressional Research Service, September 15, 2008). Their common stock, which averaged $56.54 and $57.94 respectively in 2007, closed at 65 and 88 cents respectively on 9/8/2008, one day after the federal government announced its conservatorship over the two firms (author's computations from Yahoo Finance historical data). Federal aid to AIG also resulted in substantial losses for stockholders.

[4] "Fact Sheet: Questions and Answers on Conservatorship," Federal Housing Finance Agency, September 7, 2008.

[5] Theoretical arguments that a firm's distress can repel current and potential stakeholders appear in Titman (1984), Titman and Wessels (1988), and Maksimovic and Titman (1991). Empirical evidence comes from Graham et al. (2016), who find that the filing of a reorganization bankruptcy raises the employee quit rate by 10–17%, and Babina (2015), who finds that unexpected industry shocks raise a worker's likelihood of leaving a more levered firm by about 25%. Both papers find that nondistressed levered firms pay large wage premia. Similarly, Brown and Matsa (2016) find that distressed firms advertise higher wages but attract fewer job applicants than nondistressed firms. Andrade and Kaplan (1998, p. 1475) find that about a third of distressed firms report trouble retaining key customers and suppliers. Hortaçsu et al. (2012) find that the used cars of distressed automakers fetch lower prices at auction, particularly for cars that have more time left on their warranties. In J.D. Power's 2009 Avoider Study, 18% of new car buyers who avoided a particular vehicle model cited concerns about the model's future as a reason (Hortaçsu et al., 2012).

[6] For instance, the firm's suppliers may have customized their assembly lines; its workers may have invested in specific human capital and relationships. In addition, even if compensated, stakeholders may find it prohibitively costly to replace the firm's services because of adverse selection. For instance, customers may be unable to buy outside warranties to replace their lost factory warranties. Workers may find it too costly to buy annuities to replace their lost pensions. Empirical evidence for some of bankruptcy's harmful effects comes from Graham et al. (2016), who find that employees suffer large wage losses after leaving bankrupt firms.

terms of trade from them.[7] This adverse response will raise the chance of a stakeholder exodus, thus weakening the policy's beneficial effects.

We show that it is possible to design a scheme of transfers that attains the first-best outcome at low or no cost. The scheme is countercyclical: the firm's stakeholders receive higher transfers when the firm is a less attractive trading partner. By leveling the stakeholders' demand or supply curve, such a scheme limits the terms of trade that the firm can profitably demand. In addition, the cheapest such scheme takes the form of floor-based run insurance: a guarantee that if a stakeholder does business with the firm, her payoff will not fall below some floor as the result of other agents' abandoning the firm. Such insurance is cheap as it pays out only when most agents leave the firm and thus few qualify for payments.

A simple outline of our model is as follows. There is a single firm and a continuum of *ex-ante* identical agents. The firm first chooses a price that an agent must pay in order to do business with the firm. This price may be positive or negative. If it is positive, the agents are customers; if negative, they are suppliers or workers. After the firm announces its price, nature realizes a random state. The agents see slightly noisy private signals of this state and decide whether or not to invest: to do business with the firm at its chosen price.[8]

An agent's payoff from investing is decreasing in the state and increasing in the proportion of others who invest. Obviously, it is also decreasing in the firm's price. The standard global games result (Carlsson and van Damme, 1993) extends to this context: there is a unique equilibrium, in which each agent invests if and only if her signal falls below some threshold. In the limit as the signal errors vanish, the agents perfectly coordinate: either all invest or none do. We refer to the latter event as a "run". If the firm demands a higher price, the agents become more reluctant to invest.[9] Their investment threshold falls: a run becomes more likely. Hence, the firm earns higher profits in good times, but is more likely to experience a run in which its profits are zero. At the firm's optimal price, the benefit of slightly higher profits in good times equals the cost of a slightly higher run risk.

This laissez-faire outcome is inefficient for two reasons. First, by demanding a higher price the firm raises the risk of a run, which hurts the agents. The firm ignores this negative externality: it demands an excessively high price, which makes a run too likely from a social perspective. Second, since we assume strategic complementarities in investment, agents who do not invest hurt those who do. Hence, even taking the firm's price as given, the agents run too often: their investment threshold is too low. That is, there are states in which, given the firm's price, the agents would be better off if a run were averted but one occurs nonetheless.

We then consider optimal policy. The government first announces a transfer that an agent will get if she invests. As in Angeletos and Pavan (2009), the transfer can depend on *ex-post* public information regarding fundamentals (the state) and aggregate activity (the proportion of agents who invest). We show that the first-best outcome can be implemented by offering stakeholders higher transfers when fundamentals are weak: when the state is high. If the stakeholders are

---

7 For instance, Babina (2015), Brown and Matsa (2016), and Graham et al. (2016) find that nondistressed firms offer lower wages than distressed firms.

8 The assumption that the agents are better informed about the state has two potential motivations. First, the state may reflect the appeal of the agents' outside option, about which they plausibly have better information than the firm. Second, there could be a delay between the time the firm's price announcement and the agents' investment decisions, during which the agents receive new information about economic fundamentals.

9 If the price is negative, a higher price is one that is closer to zero. For instance, if the agents are workers, the price is the negative of their wage. A higher price then corresponds to a lower wage, which makes the agents less willing to sell their labor to the firm.

consumers, such transfers raise the agents' willingness to pay relatively more in bad times, thus flattening their demand curve. The firm responds by offering a price that is low enough to entice the agents to buy from the firm in bad times as well as good: the risk of a run falls. This avoids the moral hazard problem, noted above, in which the firm exploits an insurance scheme by raising its price.

An analogous policy works when the agents are workers. Evidence in Babina (2015), Brown and Matsa (2016), and Graham et al. (2016) suggests that workers require higher wages to be willing to work at distressed firms. In the context of our model, this means that the workers' reservation wage is higher in bad states. The optimal policy equates the workers' reservation wage across states by giving higher transfers in bad states. The firm then has an incentive to offer this constant reservation wage, thus doing business in both good times and bad.

We show also that the cheapest way to level the agents' demand or supply curve is through run insurance: an agent who invests is (partly or fully) compensated for the harm she suffers when others abandon the firm. In the small-noise limit, either all agents invest or none do, so such insurance is asymptotically costless. Outside of the limit, run insurance is cheap but not costless as partial runs sometimes occur. In order to minimize costs in this case, the run insurance should be *floor-based*: an agent who invests is guaranteed that her payoff will not fall below some state-dependent floor as the result of a run. This approach is economical since payments are made only in relatively severe runs, when few agents invest and thus few qualify for payments.

As noted, run insurance is costless in the small-noise limit. However, its effect on the agents' incentives does not vanish. Why? The reason lies in the theory of global games. Such games were first studied by Carlsson and van Damme (1993) in the context of 2-player, 2-action games with two pure Nash equilibria. They showed that if, instead of the game's payoffs being common knowledge, each player receives a slightly noisy signal of these payoffs, there is a unique equilibrium. This result has been generalized to multiple players and actions, and to more general information and payoff structures (e.g., Frankel et al., 2003; Morris and Shin, 2000b, 2006). Similar findings are obtained in dynamic games with frictions and shocks under common knowledge of payoffs (Burdzy et al., 2001; Frankel and Pauzner, 2000).[10]

In a global game, as the fundamental crosses a given threshold, aggregate behavior changes abruptly. This property makes global games useful for studying aggregate fluctuations and crises. Applications include bank runs and international contagion (Goldstein and Pauzner, 2004, 2005), currency crises, debt pricing, and market crashes (Morris and Shin, 1998, 2004a, 2004b), search-driven business cycles (Burdzy and Frankel, 2005), investment cycles (Chamley, 1999; Oyama, 2004), neighborhood tipping (Frankel and Pauzner, 2002), and merger waves (Toxvaerd, 2008).

The investment subgame played by the agents in our model is a global game. In it, an agent invests if and only if her signal falls below a certain threshold. In the limit as the signal errors vanish, this threshold is the state at which an agent is indifferent on the counterfactual "Laplacian" belief that the proportion who invest will be uniform on the unit interval.[11] Hence this

[10] For limitations on the uniqueness result, see Angeletos et al. (2006), Angeletos and Werning (2006), Chassang (2010), Hellwig et al. (2006), and Morris and Shin (2004b).

[11] Kim (1996) was the first to show that in a global game informational setting with two actions and multiple agents, an agent chooses the action that is a best response under the counterfactual belief that all proportions who choose that action are equally likely. An intuition appears in Morris and Shin (2000b, pp. 61–63). In a more general informational setting, other actions may be selected (Weinstein and Yildiz, 2007). However, experiments support Kim's (1996) prediction even in settings where payoffs are common knowledge (Heinemann et al., 2009).

threshold can be manipulated by using transfers to change an agent's payoffs during partial runs, in which some agents invest but others do not. For this reason, the effect of run insurance on an agent's incentive to invest does not vanish as the signal errors shrink.

That being said, there are cases in which run insurance is too weak to fully correct the inefficiencies of the laissez-faire outcome. In this case, the policymaker must also pay the agents when all invest. Such payments have a positive asymptotic cost, which we characterize.

A few prior papers have considered policy interventions in a global games setting. In these papers the only large player is the policymaker, so there is no monopoly pricing distortion to correct. In one such paper, Angeletos and Pavan (2009) show that the first-best outcome can be obtained using a scheme of transfers which, like ours, depends on *ex-post* information about fundamentals and aggregate activity. However, their underlying complete information game has a unique equilibrium. As a result, transfers are made even in the limit as the signal errors vanish, unlike run insurance in our setting.

In related work, Angeletos et al. (2006) and Angeletos and Pavan (2013) study a game of regime change with a policymaker who sees the state and then can devote costly effort to raising the agents' cost of attacking the regime.[12] As her effort may reveal some of her information, there can be multiple equilibria. In contrast, our policymaker is uninformed when she designs her policy and the policymaker's action is an infinite-dimensional function rather than a one-dimensional effort choice.

The firm in our model can also be interpreted as a seller of a network good such as a computer platform. In this setting, the agents may represent end-users or programmers who must decide whether or not to adopt the firm's platform. When payoffs are common knowledge, this collective action problem displays multiple equilibria (Farrell and Katz, Fall/Winter 1998). We instead assume that the consumers play a global game, so the equilibrium is unique. Similarly, Argenziano (2008) and Jullien and Pavan (2016) use global games techniques to study the interaction between two firms that sell competing platforms. These papers do not consider optimal interventions.

In related work, Weyl (2010) shows the potential of "insulating tariffs" to eliminate the multiplicity of equilibria that Farrell and Katz (Fall/Winter 1998) identify. In his setting, fundamentals are common knowledge and a firm can charge a price that depends on the proportion of agents who adopt its platform. By charging a lower price when fewer buy, the firm eliminates multiplicity and attains the efficient outcome. In our context, insulating tariffs are not efficient since the firm does not internalize spillovers among agents who choose the outside option (see section 3.1).

We also study some other extensions to our base model. In section 3.2, we show that a cap on the firm's price cannot, by itself, attain the first-best. However, it can always be used in concert with run insurance to attain the first-best at zero asymptotic cost if run insurance alone is too weak to do so. Section 3.3 shows that if investors can be taxed as well as subsidized, the first-best can always be implemented with a scheme that makes no payments in the limit as the noise vanishes. In section 3.4, we show that our results are robust to certain types of noise in the policymaker's information.

The base model and the extensions discussed thus far focus on the limit in which the agents' signal errors vanish. In section 3.5, we show how to implement the first-best outcome when signal errors are large. Under a weak distributional assumption, the benefits of run insurance

[12] As the authors note, this effort choice can be interpreted as a choice among a menu of discrete policies, some of which are more effective in preventing change yet more costly for the policymaker.

carry over: such insurance is cost-minimizing if it suffices to attain the first-best. If not, the policymaker offers full run insurance as well as a subsidy to agents when all invest at certain states.

In the small-noise case, all forms of run insurance are equally costless since partial runs occur vanishingly often. Since partial runs do occur in the large-noise case, some types of run insurance are cheaper than others. We show that the cheapest run insurance is floor-based: the policymaker ensures that an investor's payoff will not fall below some floor value in a run. Intuitively, the payments in such a scheme occur mainly when fewer agents invest and thus fewer qualify for payments.

Two other extensions are also studied. One is a dynamic model in which agents have a choice of when to invest and participants learn about fundamentals over time. The other is a model of duopoly competition in which two firms compete for a common set of agents. In each case, we characterize the first-best outcome and show how to design a scheme that attains it at a minimum cost.

This paper also makes a technical contribution to the theory of global games. Past studies have assumed state monotonicity: an agent's relative payoff from investing is monotonic in the state (e.g., Morris and Shin, 2000b, Proposition 2.2, p. 67). This property is violated by our insurance schemes, so we show uniqueness under weaker assumptions (Theorem 2, section 2.3).

The rest of this paper is as follows. We present and solve the base model in section 2. Extensions are studied in section 3. Concluding remarks appear in section 4. Section 5 contains proofs of the results of section 2, while the results of section 3 are proved in our online appendix (Frankel, 2017).

## 2. Base model

There is a single firm and a unit measure of agents. All participants are risk-neutral and fully rational. The firm first publicly announces a price $p \in \Re$. Each agent $i \in [0, 1]$ then sees a private signal $x_i = \theta + \sigma\varepsilon_i$ of an exogenous random state $\theta$, where $\sigma > 0$ is a scale factor that will be taken to zero.[13] The noise terms $\varepsilon_i$ (which are independent of each other and of $\theta$) are identically distributed with continuous density $f$, cumulative distribution function $F$, and connected support contained in $\left[-1/2, 1/2\right]$. The state $\theta$ has distribution $\Phi$, with continuous and bounded density $\phi$ and support on the whole real line.

On seeing the price and their signals, the agents decide simultaneously whether or not to do business with the firm: to "invest". We refer to an agent who invests as an *investor* and to the proportion $\ell \in [0, 1]$ of agents who invest as the *investment rate*. An investor's payoff, gross of the price $p$, is denoted $v_\theta^\ell \in \Re$. A noninvestor receives an outside option payoff of $o_\theta^\ell \in \Re$.[14] An agent's *relative payoff* from investing, gross of the price $p$, is thus $r_\theta^\ell = v_\theta^\ell - o_\theta^\ell$. Her net (net relative) payoff from investing is $v_\theta^\ell - p$ (resp., $r_\theta^\ell - p$). The firm's realized payoff is $(p - c)\,\ell$ where $c \in \Re$ is a fixed parameter. We assume the firm always charges a price $p \geq c$ as any such price weakly dominates any price below $c$. We also assume the firm's price cannot exceed some finite upper bound $\overline{p} \in (c, \infty)$.

The interpretation of the constants $\overline{p}$ and $c$ depends on their sign. If they are nonnegative, the agents are the firm's potential customers, $p$ is the price it demands from them, $c \geq 0$ is the firm's

13 The case of a fixed, positive $\sigma$ is studied in section 3.5.

14 In this notation, $\ell$ represents the proportion who invest in the *firm*. Thus, $o_\theta^\ell$ is an agent's outside-option payoff when a proportion $1 - \ell$ of the others invest in the outside option.

constant marginal cost, and $\overline{p} > c$ may represent either an agent's wealth or a price that is high enough to entice an entrant to capture the entire market. If $\overline{p}$ and $c$ are nonpositive, the agents are the firm's potential workers or suppliers, $-p$ is the payment it offers them, the lower bound $-\overline{p}$ on this payment is a worker's subsistence wage or a supplier's average variable cost, and the upper bound $-c$ is the firm's marginal revenue product (assumed constant) from its best use of the agent's input.

To avoid clutter, we normalize $c$ to zero. More precisely, let $p' = p - c$ denote the firm's markup and let $\overline{p}' = \overline{p} - c$ be its maximum markup. Let $v_\theta'^\ell = v_\theta^{\ell_\theta} - c$ denote the absolute joint surplus created by an agent's choice to invest and let $r_\theta'^\ell = v_\theta'^\ell - o_\theta^\ell = r_\theta^\ell - c$ denote the relative joint surplus that is created by this decision. Finally, drop all the primes. In the new notation, an agent's net (resp., net relative) payoff from investing is once again $v_\theta^\ell - p$ (resp., $r_\theta^\ell - p$) and the outside option is still $o_\theta^\ell$, but the firm now chooses a markup $p$ from $[0, \overline{p}]$ and receives the payoff $p\ell$: the parameter $c$ has been eliminated. For simplicity, we will refer to the markup $p$ as the firm's price.

We assume the following properties of the agents' payoffs. The first property is strategic complementarities in investment:

**AM. Action Monotonicity.** There is a constant $k_1 \in (0, \infty)$ such that for any state $\theta$ and investment rates $\ell' > \ell$, $0 \leq r_\theta^{\ell'} - r_\theta^\ell \leq k_1$.

Under this property, the relative payoff function $r_\theta^\ell$ may be locally constant in the investment rate $\ell$, and may also jump upwards at certain "threshold" investment rates. For instance, investors may care whether the firm survives which, in turn, occurs only if enough agents invest.

Let

$$R_\theta = \int_{\ell=0}^{1} r_\theta^\ell d\ell \tag{1}$$

denote the mean relative payoff over all investment rates $\ell$; it is assumed differentiable in the state $\theta$. Our second assumption is that the mean complementarity is positive: at each state $\theta$ there is a positive gap between the highest relative payoff $r_\theta^1$ and $R_\theta$.

**PMC. Positive Mean Complementarities.** There is a constant $k_2 \in (0, \infty)$ such that for any state $\theta$, $r_\theta^1 - R_\theta > k_2$.

We interpret an increase in the state $\theta$ as an exogenous shock that makes the outside option more appealing. Hence, for any given investment rate $\ell$, an increase in the state $\theta$ lowers an agent's incentive to invest in the firm:

**SM. State Monotonicity.** There are constants $0 < k_3 < k_4 < \infty$ such that for every pair of states $\theta' > \theta$ and each investment rate $\ell$, $\frac{r_{\theta'}^\ell - r_\theta^\ell}{\theta' - \theta} \in (-k_4, -k_3)$.

Heinemann et al. (2009) find that experimental subjects play in accordance with the predictions of global games even when a game's payoffs (which in our model correspond to the state $\theta$) are common knowledge. Under common knowledge, State Monotonicity has no bite since it refers to states that are commonly known not to be the true state. Hence, our model may still have predictive power when this assumption does not hold.

Let

$$s_\theta = v_\theta^1 - o_\theta^0 \tag{2}$$

denote the *marginal social benefit* of investment at the state $\theta$: the social benefit if the agents choose *en masse* to invest at $\theta$.[15] We assume the maximum relative payoff exceeds this social benefit:

$$r_\theta^1 > s_\theta. \tag{3}$$

As $r_\theta^1 = v_\theta^1 - o_\theta^1$ and by (2), this is equivalent to assuming $o_\theta^0 > o_\theta^1$: the outside option is more attractive when all choose it than when none do. Intuitively, some agents who shun the firm may choose the same alternative firm, which also displays positive spillovers.

On the other hand, some agents who shun the firm are likely to choose different alternatives. Thus, we assume that the outside option's spillovers are *weaker* than those of the firm itself. More precisely, we assume that the gap between an agent's enjoyment of her choice when all others choose it, and her mean enjoyment over all investment rates $\ell$, is greater when the choice is the firm than when it is the outside option:

$$v_\theta^1 - \int_{\ell=0}^{1} v_\theta^\ell d\ell > o_\theta^0 - \int_{\ell=0}^{1} o_\theta^\ell d\ell. \tag{4}$$

By (1) and (2), (4) implies that the marginal social benefit exceeds the mean relative payoff:

$$s_\theta > R_\theta. \tag{5}$$

Our motivation for State Monotonicity is that an increase in the state $\theta$ captures exogenous changes that make the outside option more appealing. Hence, we assume that the marginal social benefit function $s$ is decreasing in $\theta$. We also assume that it is continuous and approaches positive (negative) infinity as the state falls (rises) without bound:

**DMSB. Decreasing Marginal Social Benefit.** The marginal social benefit $s_\theta$ is continuous and decreasing in the state $\theta$ and satisfies $\lim_{\theta\to-\infty} s_\theta = \infty$ and $\lim_{\theta\to\infty} s_\theta = -\infty$.

DMSB implies that $s$ equals zero at a unique, finite state $\theta^*$. At states below (above) $\theta^*$, it is socially optimal for all agents (not) to invest. Thus, $\theta^*$ is the socially optimal investment threshold.

### 2.1. Solving the base model

We now solve the above model, focusing on the limit as the noise scale factor $\sigma$ goes to zero. To ease understanding, we will skip over some technical issues that are related to how participants' behavior converges in this limit. These issues are treated formally in section 2.3.

By AM and SM, for any price $p$, the subgame played by the agents is a global game with a continuum of players and two actions. In the small-noise limit of such a game, an agent invests

[15] As noted above (footnote 14), $o_\theta^0$ is an agent's payoff from choosing the outside option when all others choose it as well: when the proportion $\ell$ who invest in the firm is zero.

at a state $\theta$ if and only if doing so is optimal under the counterfactual belief that the investment rate $\ell$ is uniform on the unit interval.[16] This holds whenever her expected net relative payoffs under this belief, $\int_{\ell=0}^{1} \left(r_\theta^\ell - p\right) d\ell$, is positive or, equivalently, if the price $p$ is less than the mean relative payoff $R_\theta$. It follows that the agents' willingness to pay at the state $\theta$ is their mean relative payoff $R_\theta$.

In the limit, the agents perfectly coordinate: if one invests, all do. Hence, at all states $\theta$ at which an agent invests (i.e., at which $R_\theta > p$), her realized relative payoff from doing so equals her maximum relative payoff $r_\theta^1$. Agents who invest at a state $\theta$ thus receive rents equal to $r_\theta^1 - p$, which is the sum of two components. The first, $R_\theta - p > 0$, is the usual rent that investors get by virtue of a firm not being able to perfectly discriminate among different states: to charge the agents' full willingness-to-pay $R_\theta$ at each state $\theta$. The second component, $r_\theta^1 - R_\theta$, which is positive by PMC, is the *strategic rent* that the agents get by virtue of the effect of strategic uncertainty on which equilibrium is selected in the investment subgame (Frankel, 2012, section 2.1.2). Our schemes work by altering these strategic rents by different amounts at different states so as to flatten the agents' demand or supply curve.

We now turn to the firm's problem. Say the firm chooses a price $p$. Since the agents' willingness to pay $R_\theta$ is decreasing in the state $\theta$ by State Monotonicity, they invest if and only if the state $\theta$ lies below the investment threshold $k$ that is given implicitly by $R_k = p$. This equality, in turn, implies a one-to-one, decreasing relationship between $p$ and $k$. So rather than choosing the price $p$, we can assume the firm directly chooses the agents' investment threshold $k$. The firm's price $p$ is then the height $R_k$ of the agents' demand curve at the state $k$. Moreover, the agents invest with probability $\Phi(k)$, where $\Phi$ is the distribution function of the state $\theta$. The firm's payoff from the threshold $k$ is thus given by the function

$$\Pi_r(k) = R_k \Phi(k). \tag{6}$$

The firm's problem now takes a familiar form. As the distribution $\Phi$ of the state is increasing, choosing an investment threshold $k$ is equivalent to choosing an expected investment probability $\Phi(k)$. This is isomorphic to the problem of a quantity-choosing monopolist, where $\Phi(k)$ is interpreted as the quantity and $R_k$ the corresponding price. The change in the price $R_k$ from increasing the quantity $\Phi(k)$ by one infinitesimal unit is then given by $\frac{dR_k}{d\Phi(k)} = \frac{dR_k/dk}{d\Phi(k)/dk} = \frac{R_k'}{\phi(k)}$. The firm's marginal revenue $\frac{d\Pi_r(k)}{d\Phi(k)}$ from this change equals $m_k = R_k + \Phi(k)\frac{R_k'}{\phi(k)}$. The derivative $R_\theta'$ is negative by State Monotonicity, so marginal revenue is less than demand: at each state $\theta$,

$$m_\theta < R_\theta. \tag{7}$$

The first-order condition for profit maximization states that at the equilibrium threshold $k$, marginal revenue $m_k$ must equal marginal cost, which we normalized to zero. The second-order condition holds as long as if marginal revenue $m_k$ is decreasing in the expected quantity $\Phi(k)$ or, equivalently, in the threshold $k$. The following assumption combines this with a limit property that ensures that $m_\theta$ crosses zero at a unique finite state $\widehat{\theta}$, which is therefore the asymptotic laissez-faire equilibrium threshold.

**DMR. Decreasing Marginal Revenue.** The marginal revenue function $m_\theta$ is continuous and decreasing in $\theta$ and satisfies $\lim_{\theta\to-\infty} m_\theta = \infty$ and $\lim_{\theta\to\infty} m_\theta = -\infty$.

---

[16] This property was first identified by Kim (1996). An intuition appears in Morris and Shin (2000b, pp. 61–63) and in our online appendix (Frankel, 2017). It is stated and proved formally in section 2.3.

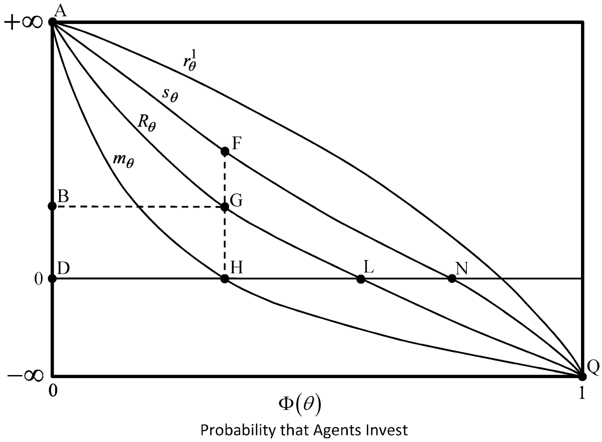


Fig. 1. Laissez-faire outcome. Vertical axis is progressively compressed (more so at bottom) so as to contain whole real line. Horizontal axis depicts agents' investment probability $\Phi(\theta)$. Social optimum is point N. Laissez-faire equilibrium is point H: price $p$ is GH, firm profits are BGHD, agent welfare is AFGB, and social welfare is AFHD. (Agent and social welfare omit fixed term $AW_F$, defined in equation (9).)

We assume that the firm's maximum feasible price $\overline{p}$ is high enough that it is never a binding constraint on the firm. A mild condition that suffices for this is

$$r^1_{\hat{\theta}} < \overline{p}, \tag{8}$$

which we henceforth assume.

We now give expressions for agent and social welfare for a general price $p$ and investment threshold $k$. When the state lies below (above) the threshold $k$, all (none) of the agents invest: realized agent welfare is $v^\ell_\theta - p$ (resp., $o^\ell_\theta$). Integrating over states, we obtain expected agent welfare $AW(k) = \int_{\theta=-\infty}^{k} \left[v^1_\theta - p\right] d\Phi(\theta) + \int_{\theta=k}^{\infty} o^0_\theta d\Phi(\theta)$ or, rearranging, $AW(k) = AW_F + \int_{\theta=-\infty}^{k} \left[s_\theta - p\right] d\Phi(\theta)$, where the fixed term

$$AW_F = \int\limits_{\theta=-\infty}^{\infty} o^0_\theta d\Phi(\theta) \tag{9}$$

represents social welfare if the agents never invest and the marginal social benefit $s_\theta$ is defined in (2). Adding agent welfare to the firm's payoff (equation (6)), we obtain social welfare $SW(k) = AW_F + \int_{\theta=-\infty}^{k} s_\theta d\Phi(\theta)$: the fixed surplus $AW_F$ plus the marginal social benefit $s_\theta$ of all agents' investing at each state $\theta$ at which they invest.

Combining (3), (5), and (7), we obtain

$$r^1_\theta > s_\theta > R_\theta > m_\theta \tag{10}$$

at every state $\theta$. These curves are depicted in Fig. 1. The vertical axis is compressed (more so near the bottom) so as to contain the whole real line. The horizontal axis depicts the investment probability $\Phi(\theta)$ at each possible threshold $\theta$. Thus, the state $\theta$ rises from left to right. The curves $r^1$, $s$, $R$, and $m$ slope downwards as these functions are decreasing in the state $\theta$.

In the laissez-faire outcome, the firm picks the point H, where its marginal revenue $m$ is zero. Its price is the height of the demand curve $R$ at this state, which is the length of the segment GH.

The firm's payoff is BGHD, agent welfare (omitting the fixed term $AW_F$), is AFGB, and social welfare (also omitting $AW_F$) is AFHD. The social optimum is at point N, where the marginal social benefit $s$ is zero.

As point H lies to the left of point N, the firm produces in too few states. There are two reasons for this. The first is the usual monopoly distortion: the firm chooses the investment threshold at which its marginal revenue $m$, rather than the agents' higher willingness to pay $R$, is zero. Intuitively, the firm ignores the negative externality that is created when, by raising its price, it induces the agents not to invest at some states where they would be willing to invest at a lower (but still positive) price. This distortion prevents investment at states between H and L.

The second source of inefficiency arises as the combination of two factors. The first is strategic rents (Frankel, 2012): because the agents face strategic uncertainty, an agent's willingness to pay is $R$ rather than her actual benefit $r^1$ from investing when all others invest. Second, by equation (4), this willingness to pay $R$ is less than the marginal social benefit $s$. Hence, there is a range of states (those between L and N) at which the marginal social benefit $s$ is positive but agents' willingness to pay $R$ is negative. At these states, even if there were no monopoly distortion – if the firm set a zero price – the agents would not invest even though they would benefit collectively from doing so.

### 2.2. *Optimal subsidy schemes*

The laissez-faire threshold $\widehat{\theta}$ lies below the efficient threshold $\theta^*$: the agents inefficiently abandon the firm at states in the interval $\left(\widehat{\theta}, \theta^*\right]$. In order to correct this inefficiency, we now consider transfer schemes in which the policymaker pays an agent a nonnegative transfer if she invests.[17] As in Angeletos and Pavan (2009), this transfer may depend on *ex-post* public information regarding fundamentals (the state $\theta$) and aggregate activity (the investment rate $\ell$). We denote the transfer as $\tau_\theta^\ell$ and will refer to the sum

$$\widetilde{r}_\theta^\ell = r_\theta^\ell + \tau_\theta^\ell \tag{11}$$

as the *augmented relative payoff function*. Let

$$\widetilde{R}_\theta = \int_{\ell=0}^{1} \widetilde{r}_\theta^\ell d\ell \tag{12}$$

denote the corresponding *augmented mean relative payoff function*: the mean of $\widetilde{r}$ over all possible investment rates $\ell$. In the presence of the subsidy scheme $\tau$, $\widetilde{R}_\theta$ is the agents' willingness to pay at the state $\theta$.

First consider full insurance against runs: at each state $\theta$ and investment rate $\ell$, the transfer $\tau_\theta^\ell$ an investor receives is equal to her full loss $r_\theta^1 - r_\theta^\ell$ from any run. By (11), her augmented relative payoff $\widetilde{r}_\theta^\ell$ is then identically equal to her maximum relative payoff from investing, $r_\theta^1$. Thus, her willingness to pay $\widetilde{R}_\theta$ equals this maximum payoff $r_\theta^1$ at every state $\theta$. The firm's profit from the threshold $k$ is then $r_k^1 \Phi(k)$ and its marginal revenue from raising the investment probability $\Phi(k)$ by one small unit is $\mu_k = r_k^1 + \Phi(k) \frac{r_k^{1\prime}}{\phi(k)}$ where $r_k^{1\prime} = \partial r_k^1 / \partial k$ (assumed here to exist) is the slope of the demand curve under full insurance. The firm's optimal threshold equates this marginal revenue to zero. In contrast, the socially optimal threshold equates the marginal social

[17] For simplicity, we do not consider transfers to noninvestors as they are not needed to attain the first best.

benefit $s$ to zero. For generic parameters, these two thresholds will differ: the outcome will be inefficient.[18]

Hence full insurance does not work in general. We now consider partial insurance. We restrict to schemes in the following class, which is formally defined in section 2.3.

**APSS (Informal).** An *Asymptotically Predictable Subsidy Scheme* (APSS) is a function $\tau_\theta^\ell$ with the following two properties.

1. Asymptotic Predictability. The augmented relative payoff function $\widetilde{r}_\theta^\ell = r_\theta^\ell + \tau_\theta^\ell$ satisfies the sufficient conditions for an asymptotically unique equilibrium in the agent subgame and, moreover, augmented demand $\widetilde{R}$ is decreasing and left-continuous.
2. No Taxation. Transfers are nonnegative: for all $\ell$ and $\theta$, $\tau_\theta^\ell \geq 0$.

Asymptotic Predictability implies that in the limit as the signal noise vanishes, either all agents invest or none do. The second condition, No Taxation, is relaxed in section 3.3. We say that an APSS is *efficient* if, in the limit as the signal noise vanishes, the firm's optimal threshold converges to the efficient threshold $\theta^*$.

Under an APSS, the augmented demand function $\widetilde{R}$ is decreasing. Hence, instead of choosing a price $p$, we may assume the firm chooses an investment threshold $k$. But the threshold does not by itself determine the price: as augmented demand $\widetilde{R}$ can now jump downwards, more than one price $p$ may yield the threshold $k$. The set of all such prices $p$ is simply the interval $\left[\lim_{\theta \downarrow k} \widetilde{R}_\theta, \lim_{\theta \uparrow k} \widetilde{R}_\theta\right]$, whose endpoints are the right and left limits of $\widetilde{R}$ at $k$. To maximize its payoff, the firm must choose the upper endpoint, which equals $\widetilde{R}_k$ as $\widetilde{R}$ is left-continuous. The firm's payoff from choosing the threshold $k$ is thus $\Pi_{\widetilde{r}}(k) = \widetilde{R}_k \Phi(k)$.

The planner's ability to lower the firm's profits at inefficient thresholds is limited in an APSS: since the transfer $\tau_\theta^\ell$ and thus its mean $T_\theta = \int_{\ell=0}^1 \tau_\theta^\ell d\ell$ are nonnegative, augmented demand $\widetilde{R}_\theta$, which equals $R_\theta + T_\theta$ by (1) and (12), cannot lie below primitive demand $R_\theta$. Hence the firm's payoff $\Pi_{\widetilde{r}}(\theta)$ from a threshold $\theta$ cannot be less than its laissez-faire payoff $\Pi_r(\theta)$ (equation (6)) from the same threshold. This laissez-faire payoff is maximized at the laissez-faire equilibrium threshold $\widehat{\theta}$. Thus, the firm will choose the efficient threshold $\theta^*$ under the scheme only if its payoff $\widetilde{R}_{\theta^*}\Phi(\theta^*)$ from doing so is not less than its laissez-faire equilibrium payoff $\Pi_r\left(\widehat{\theta}\right)$. Dividing both sides by $\Phi(\theta^*)$ implies a lower bound on augmented demand at the efficient threshold $\theta^*$:

$$\widetilde{R}_{\theta^*} \geq p_m \tag{13}$$

where we refer to

$$p_m = \Pi_r\left(\widehat{\theta}\right) / \Phi\left(\theta^*\right) \tag{14}$$

as the "minimum price" that the firm must get at the efficient threshold $\theta^*$.

We now show that an efficient scheme always exists. This scheme is asymptotically costless if the minimum price $p_m$ is less than the maximum relative payoff $r_{\theta^*}^1$ at the efficient threshold. Intuitively, the scheme in this case relies entirely on run insurance, whose cost vanishes in the small-noise limit as partial runs never occur. If the given condition does not hold, run insurance

[18] For instance, suppose $r_{\theta^*}^1 - s_{\theta^*}$ is close to zero. (By the discussion preceding equation (3), this holds if $o_{\theta^*}^0 - o_{\theta^*}^1$ is small: if the outside option displays weak spillovers at the efficient threshold $\theta^*$.) Then since $r_{\theta^*}^{1\prime}$ is negative by SM, $\mu_{\theta^*}$ is less than $s_{\theta^*}$ whence, as both functions are decreasing in the state, the equilibrium threshold will be less than the socially optimal threshold $\theta^*$: the agents will still invest too infrequently.

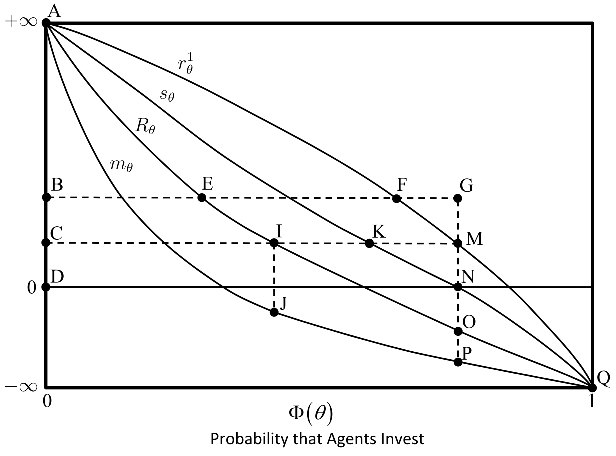


Fig. 2. An optimal APSS. Case 1: minimum price $p_m$ is smaller than $r_1^* =$ CD. APSS yields demand curve AIMOQ and marginal revenue curve AJIMPQ. Firm chooses point M, receiving payoff CMND. Omitting fixed term $AW_F$, agent welfare is AKC minus KMN and social welfare is AND. Case 2: $p_m \geq r^1_{\theta*} =$ CD. Price is BD $= p_m + \iota$ for small $\iota > 0$. Demand curve is AEGOQ; firm chooses point G. Firm payoff is BGND. Omitting $AW_F$, social welfare is AND and agent welfare is AND less BGND. Revenue cost of policy is FGM.

must be supplemented by paying the agents when all invest. In this case, we give a tight lower bound on the cost of an efficient scheme.

**Theorem 1.** *Assume the primitive relative payoff and social benefit functions, r and s, satisfy AM, SM, PMC, DMSB, and DMR, as well as equations (8) and (10).*

1. *If $p_m < r^1_{\theta*}$, then an efficient APSS exists that has zero asymptotic cost.*
2. *If $p_m \geq r^1_{\theta*}$ then (a) for each $\varepsilon > 0$, there is an efficient APSS $\tau^\varepsilon$ whose asymptotic cost is below $C(p_m) + \varepsilon$ where*

$$C(p_m) = \int\limits_{\theta \leq \theta*} \max\left\{0, p_m - r^1_\theta\right\} d\Phi(\theta),$$

*and (b) there is no efficient APSS whose asymptotic cost is below $C(p_m)$.*

**Proof.** Section 5. □

Fig. 2 illustrates the effect of the efficient APSS on the agents' demand curve in the two cases. We begin with case 1. To the right of the socially optimal point N, no transfers are given: the augmented demand curve $\widetilde{R}$ coincides with segment OQ of the original demand curve $R$. At state $\theta^*$, full insurance is given ($\tau^\ell_{\theta*} = r^1_{\theta*} - r^\ell_{\theta*}$), so $\widetilde{R}_{\theta*}$ jumps up to equal the maximum relative payoff $r^1_{\theta*}$ (point M). At states $\theta < \theta^*$, which lie to the left of point M, the insurance is gradually phased out so that the augmented demand function $\widetilde{R}$ rises as $\theta$ falls at some small positive rate $k'_3$.[19] By taking $k'_3$ to be arbitrarily small, we can guarantee that the augmented

[19] The presence of the positive constant $k'_3$ is needed to ensure that the investment subgame has a unique equilibrium. See section 2.3 for details.

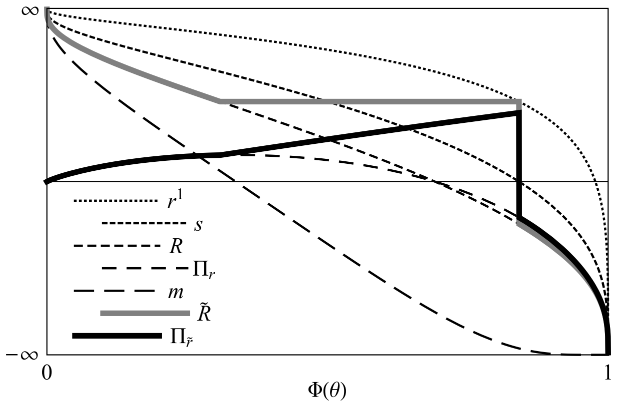


Fig. 3. A computed example that satisfies the inequality of case 1 in Theorem 1. Parameters are $\theta \sim N(1, 1)$, $k_3' = 0$, $v_\theta^\ell = 2\ell$ and $o_\theta^\ell = \theta - \ell$.

demand curve is nearly horizontal: it is depicted as segment IM in Fig. 2. The mean transfer $T_\theta$ reaches zero at point I, where $\widetilde{R}_\theta = R_\theta$. No insurance is given to the left of point I, so the augmented demand curve coincides with segment AI of the original demand curve $R$.

With this scheme, the augmented demand curve $\widetilde{R}$ is the curve AIMOQ. The corresponding marginal revenue curve is the curve AJIMPQ. The firm will choose point N if its payoff from doing so – area CMND – exceeds its payoff from the laissez-faire outcome (area BGHD in Fig. 1). This is equivalent to the condition $p_m < r_{\theta*}^1$ which holds by assumption in case 1. Hence the firm gets CMND and social welfare rises to AND. Agent welfare is AKC minus KMN. (As before, agent and social welfare omit the fixed term $AW_F$.)

Fig. 2 also depicts case 2, when the minimum price $p_m$ exceeds CD. Here, the length of segment BD equals the minimum price $p_m$ plus a small increment $\iota$ that ensures that the firm strictly prefers to choose point G. The demand curve is now AEGOQ. The asymptotic cost of the policy is the area of segment FGM, which converges to a lower bound of $C(p_m)$ as the increment $\iota$ and the slope $-k_3'$ of the segment BG both go to zero.

Fig. 3 shows a computed example of case 1. The computed profit functions $\Pi_r$ and $\Pi_{\tilde{r}}$ also appear. The parameters are $\theta \sim N(1, 1)$, $k_3' = 0$, $v_\theta^\ell = 2\ell$ and $o_\theta^\ell = \theta - \ell$, whence $r_\theta^\ell = 3\ell - \theta$, $R_\theta = 3/2 - \theta$, and $s_\theta = 2 - \theta$. Augmented demand $\widetilde{R}$ is the thick gray curve and the firm's augmented payoff function $\Pi_{\tilde{r}}$ is the thick black curve. The scheme leads the firm to lower its probability of failure from $1 - \Phi\left(\widehat{\theta}\right) = 0.66$ to $1 - \Phi(\theta^*) = 0.16$.[20] In this figure, the scheme is phased out at the maximum possible rate: augmented demand $\widetilde{R}$ is level during the phase-out.[21]

## 2.3. Formal results

Sections 2.1 and 2.2 implicitly assume that, as the signal errors shrink, the agents' choices converge to the Laplacian action and the firm's optimal threshold converges to a maximizer of its

[20] A large initial failure probability is chosen in order to avoid all of the "action" occurring in a small area on the right side of the figure.

[21] A decrease in the phase-out rate (which corresponds to an increase in $k_3'$) would cause the straight segments of the thick gray and black curves to rotate clockwise, with their right endpoints remaining stationary. The straight segment of the thick black curve would also become concave.

asymptotic profit function. Moreover, the definition of an APSS in section 2.2 does not specify which conditions on $\widetilde{r}$ suffice for an asymptotically unique equilibrium in the agent subgame. In this section we fill these gaps and precisely specify the efficient scheme.

As an agent's relative payoff function may be either $r$ or $\widetilde{r}$, we define our payoff properties in terms of a general relative payoff function $h_\theta^\ell$ with corresponding mean

$$H_\theta = \int_{\ell=0}^{1} h_\theta^\ell d\ell. \tag{15}$$

The notation A$(h)$ will mean that $h$ satisfies the property A. For instance, SM$(h)$ will mean that $h$ satisfies State Monotonicity.

While $r$ satisfies AM, PMC, and SM, the efficient scheme will yield an augmented relative payoff function $\widetilde{r}$ that violates PMC and SM. However, it satisfies the following two weaker properties which, together with AM, suffice for uniqueness.[22] First, if the relative payoff ever *increases* in the state, it does so in a bounded way:

**OSL$(h)$. One-Sided Lipschitz Continuity.** There is a constant $k_5 \in (0, \infty)$ such that for all investment rates $\ell$ and states $\theta' > \theta$, $h_{\theta'}^\ell - h_\theta^\ell < k_5 \left(\theta' - \theta\right)$.

Second, the mean relative payoff function $H$ is left-continuous, satisfies the upper bound $-k_3$ in SM, and satisfies the lower bound $-k_4$ at all points of continuity:

**MSM$(h)$. Mean State Monotonicity.** For any states $\theta'$ and $\theta$ such that $\theta' > \theta$, $\frac{H_{\theta'} - H_\theta}{\theta' - \theta} < -k_3$. Moreover, $H$ is left-continuous and, if $H$ is continuous throughout the interval $\left(\theta, \theta'\right)$, then $\frac{H_{\theta'} - \lim_{\theta'' \downarrow \theta} H_{\theta''}}{\theta' - \theta} > -k_4$.

**Claim 1.** *If a relative payoff function h satisfies SM, then it also satisfies MSM and OSL.*

**Proof.** Trivial. $\square$

Fix a price $p$, and let

$$\theta_H^p = \sup\{\theta : H_\theta \geq p\} = \inf\{\theta : H_\theta \leq p\} \tag{16}$$

denote the boundary between states at which the mean relative payoff $H_\theta$ exceeds and is less than $p$.[23] We have the following result.

**Theorem 2** *(Agent subgame). Let the agents' relative payoff function be $h$. Assume AM$(h)$, MSM$(h)$, and OSL$(h)$ and let $S$ be any bounded subset of $\Re$. Then for any $\varepsilon > 0$ there is a $\delta > 0$ such that for any price $p \in S$ and any private noise scale factor $\sigma$ in the interval $(0, \delta)$, in any strategy profile that survives iterated deletion of strictly dominated strategies, each agent invests if her signal is less than $\theta_H^p - \varepsilon$ and does not invest if her signal exceeds $\theta_H^p + \varepsilon$, where $\theta_H^p$ is defined in (16).*

[22] Technically, AM and SM suffice to show uniqueness when payoffs are given by $r$. The assumption that $r$ satisfies PMC is used, instead, to show that $\widetilde{r}$ satisfies AM – a property that plays a critical role in the uniqueness argument for $\widetilde{r}$.

[23] The sup and inf are equal by MSM, which we will assume $h$ satisfies.

**Proof.** Section 5. □

Theorem 2 implies that the agents' asymptotic willingness to pay at the state $\theta$ is $H_\theta$. Why? By MSM, at states $\theta$ below (above) the boundary $\theta_H^p$ defined in (16), the mean relative payoff function $H_\theta$ is greater (less) than the price $p$. And Theorem 2 implies that in the limit as the signal errors shrink, agents (do not) invest at states $\theta$ below (resp., above) the boundary $\theta_H^p$. Hence, agents invest at the state $\theta$ in the limit if and only if $p$ is less than $H_\theta$, which therefore must equal their asymptotic willingness to pay at $\theta$.

We now turn to the firm's problem in the limit as the signal noise vanishes, when the agents' relative payoff function $h$ satisfies the assumptions of Theorem 2: AM, MSM, and OSL. By MSM, the agents' asymptotic willingness to pay $H_\theta$ is decreasing in $\theta$, so instead of choosing a price $p$ we may assume the firm chooses a state $k$ below which the agents invest. The optimal price that yields the threshold $k$ is then $H_k$ as $H$ is left-continuous by MSM. The firm's asymptotic expected payoff is thus

$$\Pi_h(k) = H_k \Phi(k). \tag{17}$$

We will now show formally that as the signal errors shrink, any threshold that is optimal for the firm must converge to a maximizer of $\Pi_h$. For each $\sigma > 0$, let $p_\sigma \in [0, \overline{p}]$ be an optimal price for the firm when the signal noise scale factor is $\sigma$. By Theorem 2, for each $\varepsilon > 0$ there is a $\delta > 0$ such that for any $\sigma \in (0, \delta)$, each agent invests if her signal is less than $\theta_H^{p_\sigma} - \varepsilon$ and does not invest if her signal exceeds $\theta_H^{p_\sigma} + \varepsilon$.[24] In this sense, the agents' investment threshold must become arbitrarily close to $\theta_H^{p_\sigma}$ as $\sigma$ shrinks. The following result shows that $\theta_H^{p_\sigma}$ in turn must converge to the maximizer of the firm's asymptotic payoff function $\Pi_h$ if a unique one exists, and cannot converge to a state that does not maximize $\Pi_h$.[25]

**Theorem 3.** *Assume AM($h$), MSM($h$), and OSL($h$).*

1. *If $\theta$ is the unique maximizer of $\Pi_h$, then $\theta_H^{p_\sigma}$ converges to $\theta$ as $\sigma$ shrinks.*
2. *If $\theta$ is not a maximizer of $\Pi_h$, then $\theta_H^{p_\sigma}$ does not converge to $\theta$ as $\sigma$ shrinks.*

**Proof.** Section 5. □

Armed with these results, we can now give the formal definition of an APSS.

**APSS (Formal).** An *Asymptotically Predictable Subsidy Scheme* (APSS) is a function $\tau_\theta^\ell$ with the following two properties.

1. Asymptotic Predictability. The augmented relative payoff function $\widetilde{r}_\theta^\ell = r_\theta^\ell + \tau_\theta^\ell$ satisfies AM, MSM, and OSL.
2. No Taxation. Transfers are nonnegative: for all $\ell$ and $\theta$, $\tau_\theta^\ell \geq 0$.

24 The limiting threshold $\theta_H^{p_\sigma}$ is the boundary between the regions where $H_\theta$ is greater (less) than $p_\sigma$. It is given by equation (16) evaluated at $p = p_\sigma$.

25 One may wonder why we do not simply show that the thresholds $\theta_H^{p_\sigma}$ must converge to a maximizer of $\Pi_h$. The reason is that if $\Pi_h$ has multiple maximizers, the thresholds $\theta_H^{p_\sigma}$ need not converge at all. Rather, they might cycle among the maximizers of $\Pi_h$ as $\sigma$ shrinks. In designing our schemes we will avoid this situation by ensuring that $\Pi_h$ has a unique maximizer.

Such a scheme is *efficient* if the firm's optimal thresholds all converge to $\theta^*$ as the signal errors shrink:

**Efficiency.** An APSS $\tau$ is efficient if, for any $\varepsilon > 0$ there is a $\delta > 0$ such that for any threshold $k$ that is optimal for the firm to choose given any noise scale factor $\sigma$ in $(0, \delta)$, $|k - \theta^*| < \varepsilon$.

We conclude this section by presenting the scheme that underlies Theorem 1. We begin with case 2.

- Case 2: $p_m \geq r^1_{\theta^*}$. Fix two arbitrarily small constants $\iota > 0$ and $k_3' \in (0, k_3)$.[26] Let $R^*_\theta$ denote the function $p_m + \iota + k_3'(\theta^* - \theta)$, which exceeds $r^1_\theta$ at $\theta = \theta^*$. As $\theta$ falls, $R^*_\theta$ rises but more slowly than either $r^1_\theta$ or $R_\theta$ by SM and since $k_3'$ is less than $k_3$. Hence it equals $r^1_\theta$ at some unique $\theta_2 < \theta^*$ and then, by PMC, equals $R_\theta$ at some unique $\theta_1 < \theta_2$[27]:

$$R^*_{\theta_2} = r^1_{\theta_2} \text{ and} \tag{18}$$

$$R^*_{\theta_1} = R_{\theta_1}. \tag{19}$$

The augmented demand function we will construct is then

$$\widetilde{R}_\theta = \begin{cases} R_\theta & \text{if} \quad \theta > \theta^* \\ \max\left\{R_\theta, R^*_\theta\right\} & \text{if} \quad \theta \leq \theta^* \end{cases} \tag{20}$$

Let $T_\theta$ equal the required mean transfer $\widetilde{R}_\theta - R_\theta$ at each state $\theta$. The transfer scheme $\tau^\ell_\theta$ is then given by

$$\tau^\ell_\theta = \begin{cases} \alpha_\theta\left(r^1_\theta - r^\ell_\theta\right) \geq 0 & \text{if} \quad \theta \in [\theta_1, \theta_2] \text{ and } \ell > 0 \\ \widetilde{R}_\theta - r^\ell_\theta \geq 0 & \text{if} \quad \theta \in \left[\theta_2, \theta^*\right] \text{ and } \ell > 0 \\ 0 & \text{if} \quad \theta \notin \left[\theta_1, \theta^*\right] \text{ or } \ell = 0 \end{cases} \tag{21}$$

where $\alpha_\theta$ denotes the ratio $\frac{T_\theta}{r^1_\theta - R_\theta} \in [0, 1]$.[28] Moreover, at states $\theta$ in $\left[\theta_2, \theta^*\right]$, $\widetilde{R}_\theta$ is not less than $r^\ell_\theta$. Hence, the transfer $\tau^\ell_\theta$ is nonnegative. It is also easy to verify that $\tau^\ell_\theta$ integrates, over investment rates $\ell$, to $T_\theta$, so augmented demand is indeed given by (20). By definition of $\theta_2$, $\widetilde{R}_{\theta_2}$ equals $r^1_{\theta_2}$ and thus $\alpha_{\theta_2}$ equals one, whence the two specifications of $\tau^\ell_\theta$ coincide at $\theta = \theta_2$: each equals $r^1_{\theta_2} - r^\ell_{\theta_2}$.
- Case 1: $p_m < r^1_{\theta^*}$. In this case the construction is as in case 2 but $\iota$ equals $r^1_{\theta^*} - p_m$, whence $\theta_2$ coincides with $\theta^*$.[29]

## 3. Extensions

We now discuss several extensions: firm-provided private insurance, a price cap, taxation, noise in the policymaker's information, and large (nonvanishing) signal errors. We then use our

[26] The precise meaning of "small" in this context is explained in the proof of Theorem 1.

[27] The states $\theta_1$ and $\theta_2$ correspond to points E and F, respectively, in Fig. 2.

[28] For $\theta \in \left[\theta_1, \theta_2\right]$, $R^*_\theta$ is in $\left[R_\theta, r^1_\theta\right]$ by construction, so $T_\theta = R^*_\theta - R_\theta$ lies in $\left[0, r^1_\theta - R_\theta\right]$; hence, $\alpha_\theta \in [0, 1]$.

[29] The states $\theta_1$ and $\theta_2$ now correspond to points I and M, respectively, in Fig. 2.

results from the cases of small and large signal errors to study a dynamic model in which participants learn about fundamentals and have a choice of when to invest. Finally, we show how to attain the first-best outcome in a duopoly model in which two firms compete for the loyalty of a common set of stakeholders. All results that are not proved in the text are proved in our online appendix (Frankel, 2017).

### 3.1. Private insurance

We first consider firm-provided private insurance by adapting Weyl's (2010) model of "insulating tariffs". In Weyl's model, the firm can commit to a schedule in which the price an agent pays depends on the proportion of agents who invest. It can also depend on the agents' payoff function, which Weyl assumes is common knowledge.

This is most easily captured in our setting by letting the firm announce not a single price but rather a price schedule $p_\theta^\ell$. The agents then see their signals and decide whether to invest as before. Once all the agents have made their decisions and the participation rate $\ell$ and state $\theta$ are publicly verified, each investor pays the firm the price $p_\theta^\ell$. As before, we focus on the small-noise limit. We also no longer normalize the parameter $c$ to zero.[30]

What sort of scheme will the firm choose? Assume an agent invests if she is indifferent. Then the firm can induce the agents all to invest at the state $\theta$ by offering a schedule in which the eventual price $p_\theta^\ell$ equals an agent's relative payoff $r_\theta^\ell$. Since all invest, the realized price is the maximum relative payoff $r_\theta^1$ at the state $\theta$. Hence the firm desires this outcome whenever $r_\theta^1$ exceeds the parameter $c$. An optimal schedule for the firm is thus one in which the price $p_\theta^\ell$ equals $r_\theta^\ell$ (resp., $c$) if $r_\theta^1$ exceeds (is less than) $c$. The agents then invest whenever $r_\theta^1$ exceeds $c$, whence the firm earns $\max\left\{r_\theta^1 - c, 0\right\}$.

Our model (unlike Weyl's) has features that make an insulating tariff unattractive. First, the outcome is inefficient: at states $\theta$ where $r_\theta^1 > c > s_\theta$, the agents invest with the firm but the outside option is the efficient choice. Intuitively, the firm ignores the positive spillovers enjoyed by those who choose the outside option.

Moreover, an insulating tariff may not be credible if the agents are suppliers or workers. In this case, Action Monotonicity implies that the more agents abandon the firm, the higher is the payment $-r_\theta^\ell$ to those who remain. But in practice, such a run might bankrupt the firm. In bankruptcy, a firm's workers and suppliers are treated as unsecured creditors and thus may not be paid at all. Hence, workers or suppliers would be justifiably skeptical about a firm's pledge to pay them more in a severe run.

### 3.2. A price cap

Since one of the distortions in the laissez-faire outcome comes from the firm charging an excessive price, one may wonder whether a price cap is a reasonable alternative to our insurance scheme. Under our assumptions, the demand curve $R$ lies below zero at the socially optimal threshold $\theta^*$ in Fig. 1. An efficient price cap would thus have to hold the firm to the negative price $-$NO in Fig. 2. Since profits from this price are negative, the firm will prefer to shut down. Thus, a price cap cannot implement the first best outcome, while an APSS can always do so. The

[30] In particular, the firm now chooses a price $p$ from the interval $\left[c, \overline{p}\right]$ where $c$ and $\overline{p}$ are positive (negative) if the agents are customers (resp., workers or suppliers). The firm earns $p - c$ per agent who invests.

advantage of an APSS is that it not only gives the firm an incentive to lower its price, but also raises the agents' willingness to pay. A price cap is inferior as it lacks the latter effect.

On the other hand, it can sometimes be cost-effective to supplement an APSS with a price cap. This occurs in case 2 of Theorem 1, when the minimum price $p_m$ needed to entice the firm to choose the efficient threshold exceeds the agents' maximum willingness to pay $r^1_{\theta*}$. Then by Theorem 1, there is no costless APSS that attains the first-best outcome: the least-cost APSS has the positive cost FGM in Fig. 2. Now suppose instead that the firm's price is capped at $r^1_{\theta*}$ and, moreover, the APSS of case 1 is imposed. In Fig. 2, this APSS yields the augmented demand curve AIMOQ (rather than AEGOQ). Since the price $r^1_{\theta*}$ is positive, the firm will charge this price and the outcome will be efficient. But unlike the APSS of case 1, this scheme is asymptotically costless. In this way, a price cap can eliminate the positive cost of attaining the efficient outcome in case 2 of Theorem 1.

### 3.3. *Taxation*

We showed in section 2 that there exists a costless efficient scheme when the minimum price $p_m$ is low enough. This scheme relies only on subsidies. With taxation, a stronger result is possible. For any minimum price $p_m$, there exists an efficient scheme in which, in the limit as the noise vanishes, no transfers are paid either to or from the agents.

We first describe, intuitively, how to use taxation to modify the APSS of case 1 in our base model in order to obtain a scheme that works in case 2 as well. Recall that this APSS uses subsidies between points I and M in Fig. 2 to yield the augmented demand curve AIMOQ. These subsidies are also present under the new policy. However, the policies differ to the left of point I: while the APSS does nothing, the new policy uses taxes on investors in order to keep the demand curve level in this region as well. The full augmented demand curve under the scheme is thus ACMOQ.

This new scheme always induces the firm to choose point M. Why? On segment CM the price $r^1_{\theta*}$, and thus the firm's payoff $r^1_{\theta*}\Phi(\theta^*)$, is positive as $r^1_{\theta*} > s_{\theta*} = 0$ by (3). And point M is better than any other point on CM since it offers the highest probability of investment. Finally, the prices on segment OQ of the demand curve are negative: the firm will not choose these points. Hence the new scheme induces the firm to choose point M: the efficient outcome is implemented.

The formal result is as follows. We consider the following class of schemes. It differs from an APSS in two ways: transfers may be negative, and no transfers are made when agents coordinate. The latter property implies that no transfers are made in the small-noise limit.

**APRS.** An *Asymptotically Predictable, Revenue-Neutral Scheme* (APRS) is a transfer function $\tau$ with the following two properties.

1. Asymptotic Predictability. The augmented relative payoff function $\widetilde{r}^{\ell}_{\theta} = r^{\ell}_{\theta} + \tau^{\ell}_{\theta}$ satisfies the sufficient conditions for an asymptotically unique equilibrium in the agent subgame and, moreover, augmented demand $\widetilde{R}$ is decreasing and left-continuous.[31]
2. No Transfers Under Coordination: for all $\theta$, $\tau^1_{\theta} = 0$.

Unlike an APSS, there always exists an efficient such scheme[32]:

[31] The conditions for asymptotic uniqueness are AM, MSM, and OSL; see section 2.3.

[32] A scheme is efficient if it leads the firm asymptotically to choose the efficient threshold; see section 2.3.

**Theorem 4.** *Assume the primitive relative payoff and social benefit functions r and s satisfy AM, SM, PMC, DMSB, and DMR, as well as equations (8) and (10). Then there always exists an efficient APRS.*

**Proof.** See Frankel (2017). □

*3.3.1. Taxation: technical details*

A precise specification of the efficient APRS with taxation is as follows. Let $R^*_\theta$ now denote the function $r^1_{\theta^*} + k'_3(\theta^* - \theta)$, which equals $r^1_\theta$ at $\theta = \theta^*$. As $\theta$ falls, $R^*_\theta$ rises but more slowly than either $r^1_\theta$ or $R_\theta$ by SM and since $k'_3$ is less than $k_3$. Hence by PMC,

$$R^*_\theta \text{ equals } R_\theta \text{ at some unique } \theta = \theta_1 < \theta^* \text{ and, moreover, } R^*_\theta \gtrless R_\theta \text{ as } \theta \lessgtr \theta_1. \tag{22}$$

We now let $\theta_0$ be low enough that, if the firm can obtain the price $r^1_{\theta^*}$ at the efficient threshold $\theta^*$, it will never choose a threshold $\theta \leq \theta_0$ even if it can get the maximum price $\overline{p}$:

$$\overline{p}\Phi(\theta_0) < r^1_{\theta^*}\Phi\left(\theta^*\right). \tag{23}$$

The augmented demand function we will construct is then

$$\widetilde{R}_\theta = \begin{cases} R_\theta & \text{if} \quad \theta \notin (\theta_0, \theta^*] \\ R^*_\theta & \text{if} \quad \theta \in (\theta_0, \theta^*] \end{cases} \tag{24}$$

which appears as the curve ACMOQ in Fig. 2. Let $T_\theta$ equal the required mean transfer $\widetilde{R}_\theta - R_\theta$ at each state $\theta$. The transfer scheme $\tau^\ell_\theta$ is then given by

$$\tau^\ell_\theta = \begin{cases} \alpha_\theta\left(r^1_\theta - r^\ell_\theta\right) \geq 0 & \text{if} \quad \theta \in \left[\theta_1, \theta^*\right] \\ 2(1-\ell)\,T_\theta < 0 & \text{if} \quad \theta \in (\theta_0, \theta_1) \\ 0 & \text{if} \quad \theta \notin (\theta_0, \theta^*] \end{cases} \tag{25}$$

where $\alpha_\theta$ again denotes the ratio $\frac{T_\theta}{r^1_\theta - R_\theta}$.[33] It is also easy to verify that $\tau^\ell_\theta$ integrates, over investment rates $\ell$, to $T_\theta$, so augmented demand is indeed given by (24). By definition of $\theta_1$, $\widetilde{R}_{\theta_1}$ equals $R_{\theta_1}$. Hence $T_{\theta_1}$ and $\alpha_{\theta_1}$ equal zero, whence the two specifications of $\tau^\ell_\theta$ coincide at $\theta = \theta_1$: each equals zero. (This occurs at point I in Fig. 2.)

*3.4. Noise in the policymaker's information*

Our model assumes the policymaker observes the state $\theta$ and investment rate $\ell$ without noise. We now show that these assumptions are largely dispensible.

We first show that the policymaker can implement the first-best outcome with knowledge of the investment rate $\ell$ alone. We make the additional mild assumption that agents always play a threshold equilibrium in the investment subgame. In this result we rely on an APRS, in which transfers occur only during partial runs.

**Claim 2.** *Assume the primitive relative payoff function r satisfies AM, SM, and PMC. Let $\tau$ be the APRS described in (25). Assume that, for any price p, the agents' decisions are given*

[33] The transfer $\tau^\ell_\theta$ has the signs claimed in (25) since, by construction, $R^*_\theta$ is in $\left[R_\theta, r^1_\theta\right]$ (resp., lies below $R_\theta$) for $\theta$ in $\left[\theta_1, \theta^*\right]$ (resp., for $\theta$ in $(\theta_0, \theta_1)$).

*by a threshold equilibrium: each agent $i$ invests if and only if her signal $x_i$ falls below some threshold $k_p$. Then for any noise scale factor $\sigma > 0$, the policymaker can implement $\tau$ without directly observing $\theta$ or any signal of $\theta$.*

**Proof.** By hypothesis, each agent $i$ invests whenever $x_i = \theta + \sigma\varepsilon_i < k_p$ or, equivalently, when $\varepsilon_i < \frac{k_p-\theta}{\sigma}$, which holds with probability $F\left(\frac{k_p-\theta}{\sigma}\right)$. This probability must then equal the proportion $\ell$ who invest by the law of large numbers. Hence $F\left(\frac{k_p-\theta}{\sigma}\right) = \ell$. If $\ell$ is either zero or one, no payments are called for under $\tau$: the policymaker does nothing. Now suppose instead that $\ell$, and thus $F\left(\frac{k_p-\theta}{\sigma}\right)$, lies in $(0, 1)$. As the support of $F$ is connected, $F$ is strictly increasing at $\frac{k_p-\theta}{\sigma}$. But then the policymaker can perfectly infer $\frac{k_p-\theta}{\sigma} = F^{-1}(\ell)$. From this and her knowledge of $k_p$, she can also infer the state $\theta$ and thus any required payment under the scheme $\tau$. □

Now suppose the policymaker sees the state $\theta$ but not the investment rate $\ell$. Assume moreover that applying for an insurance payment is costless. Then if these payments are nonnegative, it is weakly dominant for all $\ell$ investors to apply. The government will then be able to infer the proportion $\ell$ by observing these applications. Accordingly, the policymaker can always implement the APSS $\tau$ defined in equation (21).[34]

Finally, suppose the policymaker sees both the state $\theta$ and the investment rate $\ell$ with noise (or not at all). If subsidies alone are used then, by the preceding argument, the policymaker can infer $\ell$ by observing how many agents apply for payments. If $\ell \in (0, 1)$, the policymaker can then infer the state $\theta$ using the technique of Claim 2. Thus, the policymaker can implement the APSS of equation (21) when $p_m < r^1_{\theta^*}$, as transfers occur only during partial runs in this case. If $p_m \geq r^1_{\theta^*}$, this APSS also involves some subsidies that are paid when all agents invest. But if all invest, the policymaker can infer only that the state lies below $\theta^*$. Hence it cannot implement the particular APSS of equation (21) in this case.

### 3.5. Large noise

The scheme in our base model (equation (21)) does not depend on the distribution $F$ of the agents' signal errors. It induces efficient behavior in the small-noise limit, but not with large noise. In Frankel (2017), we show how to exploit the noise structure $F$ in order to induce efficient behavior with large noise.[35] In this efficient outcome, partial runs occur with positive probability since the agents see different signals. Hence run insurance is no longer costless. We show that, under a mild distributional assumption, the cheapest type of run insurance is *floor-based*: an investor's payoff is guaranteed not to fall below some floor level in a run. Such schemes are cheap because they concentrate their subsidies in situations when fewer agents invest and thus fewer qualify for subsidies.

[34] In addition, our schemes work by insuring the agents against the harm they suffer when $\ell$ declines. Often this harm is the result of large, publicly observable events such as the firm's liquidation. In this case, insuring the agents (in a state-dependent way) against such adverse events is equivalent to insuring them against declines in $\ell$. Thus, the policymaker can implement the scheme without observing $\ell$ itself if it sees such payoff-relevant events (together with the state $\theta$).

[35] In addition to assuming the planner knows $F$, these schemes also rely on two new assumptions: the state is uniformly distributed and the agents play a threshold equilibrium if one exists. In contrast, section 2 allows a general prior and relies only on iterative strict dominance.

An outline of our approach is as follows. We first show that Figs. 1 and 2 apply also to the large-noise case if we interpret the horizontal axis as the cumulative distribution of the agents' signals rather than of the state. We then exploit this fact to derive an efficient floor-based scheme, which yields the augmented demand curve AIMOQ or AEGOQ (depending on parameters) in Fig. 2. Finally, we show that when the signal error distribution $F$ is log-concave,[36] this floor-based scheme is cost-minimizing in the class of efficient schemes. For details the reader is referred to Frankel (2017).

### 3.6. Learning about the state

In Frankel (2017), we also study a two-period model in which the agents learn about the state and have a choice of when and whether to invest in a relationship with the firm. Relationships are long-lasting: if an agent invests in a relationship with the firm in period 1, she continues to enjoy this relationship in period 2.[37] Since the firm also learns about the state, the agents face a tradeoff. If they wait, they can base their decision on more precise knowledge of the state. On the other hand, the firm learns about the state as well and thus can better extract the agents' informational rents. Waiting until period 2 also shrinks an agent's strategic rents (Frankel, 2012): since investment is irreversible, in period 2 there are fewer agents whose investment decisions are in doubt.

As an example one might consider a new, unknown firm, or an existing firm that suddenly experiences rapid growth for unknown reasons. For instance, as current tech giants such as Microsoft, Google, and Amazon first grew, academic researchers may first have hesitated to take jobs there because of uncertainty (shared by the firms) about job security and the research environment. Hence the firms had to offer high salaries to their first hires. As more and more researchers joined on, both types of risks declined, which let the firm make lower offers.

Our main result is that if taxes cannot be levied, the efficient outcome can be implemented at a minimum cost by using a floor-based subsidy scheme in period 1, followed by a floor-based subsidy scheme in period 2 that depends on the period-1 outcome. For details the reader is referred to Frankel (2017).

### 3.7. Duopoly competition

Our final extension is a model of two firms who compete for agents. An example is a duopoly in computer platforms, in which each firm competes for customers and/or talented workers. The state is now a shock to an agent's relative payoff from doing business with one firm vis-a-vis the other. As explained below, the scheme of our base model will not work in this setting. However, a different scheme can be constructed that is asymptotically efficient. While this scheme relies on taxation, no transfers are made in the limit as the signal noise vanishes (as in the model of section 3.3). Details of the scheme appear in our online appendix (Frankel, 2017).

The model is constructed from our base model (section 2) by assuming that the outside option consists of investing in a competing firm. Let us refer to the original firm as firm 1 and to the

[36] This means that $\ln F(\varepsilon)$ is a concave function of the signal error $\varepsilon \in [-1/2, 1/2]$ or, equivalently, that $f(\varepsilon)/F(\varepsilon)$ is nonincreasing in $\varepsilon$. An example is the uniform distribution on $[-1/2, 1/2]$.

[37] For instance, a worker may have to relocate to the city in which the firm is located; a supplier may need to customize its assembly line to produce inputs that are tailored to the firm's needs; a platform user may need to purchase and learn to use the firm's software.

outside option as firm 2. The firms have the same marginal cost which, as above, is normalized to zero. Firms 1 and 2 first simultaneously set their prices $p_1$ and $p_2$, respectively. The agents then see their signals and choose in which firm to invest.[38] If a proportion $\ell$ invest in firm 1 at state $\theta$, investors in firm 1 (resp., 2) get the payoffs $v_\theta^\ell - p_1$ (resp., $o_\theta^\ell - p_2$). Thus, the relative payoff from investing in firm 1 vs. firm 2 is $r_\theta^\ell - (p_1 - p_2)$ rather than, in the monopoly case, $r_\theta^\ell - p$: from the agents' point of view, the price differential $p_1 - p_2$ now plays the role of the monopoly price $p$.

Now assume $r$ satisfies the primitive assumptions AM, PMC, and SM. Since each firm's price lies in $\left[0, \overline{p}\right]$, the price differential $p_1 - p_2$ lies in the compact set $\left[-\overline{p}, \overline{p}\right]$. Thus, by Claim 1 and Theorem 2, in the limit as the private noise scale factor $\sigma$ shrinks, each agent invests in firm 1 if her signal is less than the threshold $k = \theta_R^{p_1 - p_2}$; else she invests in firm 2. By (16) and the continuity of the mean payoff function $R$ (which is implied by SM), the threshold $k$ is uniquely defined by

$$R_k = p_1 - p_2. \tag{26}$$

Thus, by SM, $k$ is a monotonic function of both $p_1$ and $p_2$: it is jointly controlled by the two firms. In equilibrium, they must prefer the same threshold $k$.

Because the threshold $k$ must be optimal for both firms, the APSS of the base model will not implement the social optimum. For suppose a scheme is in place that yields the augmented demand curve AIMOQ in Fig. 2. Suppose the firms choose the efficient point, M. The probability that the agents invest in firm 2 is $1 - \Phi(\theta)$, which equals the horizontal distance between point M and the right boundary of the box. By the preceding discussion, the height of the augmented demand curve $\widetilde{R}$ equals the difference in prices $p_1 - p_2$. Suppose now that firm 2 lowers its price slightly. The outcome will move from point M to point I. This raises firm 2's success probability $1 - \Phi(\theta)$ by the length of segment IM. As long as $p_2$ was initially positive, firm 2's profits will rise. Since firm 2 has a profitable deviation, point M is not an equilibrium under the given scheme.

In Frankel (2017) we construct an alternative scheme, which is asymptotically efficient and revenue-neutral. The scheme consists of a simple state-dependent "miscoordination tax" on each firm's investors. That is, in order to raise an agent's relative willingness to invest in, say, firm 1 at a given state, we impose a tax on agents who choose firm 2. In order to preserve Action Monotonicity, this tax is decreasing in the proportion of agents who choose firm 2.[39] The reader is referred to Frankel (2017) for details.

## 4. Conclusions

Evidence shows that distressed firms have trouble retaining the loyalty of their small stakeholders: their customers, suppliers, and workers. If these stakeholders abandon the firm *en masse* ("run"), the firm will have to be liquidated. If a firm is very large or highly interconnected with other firms, policymakers may be tempted to bail the firm out. Such bailouts can have a large tax revenue cost, and can be politically risky.

To avoid costly bailouts, policymakers have tried to insure stakeholders who invest against the risk that others will not. However, complete insurance of this type leads to moral hazard in which

[38] We assume an agent must invest in one firm or the other.

[39] While one may also attain the first-best while relying only on subsidies, this is only so for some parameters and the schemes are more complex. In contrast, there is always a simple and efficient tax-based scheme.

the firm demands better terms of trade from its stakeholders. These demands make the firm more vulnerable to a run, thus blunting the effects of the insurance.

We show that the efficient outcome can be attained through a scheme of partial, state-dependent insurance. More generous insurance is given in bad states so as to level the stakeholders' demand or supply curve. The scheme can be designed so as to induce the firm to select the first-best run risk: the outcome is efficient. A graphical intuition shows that the situation is analogous to the case of a monopolist which, facing perfectly inelastic demand, selects the efficient output quantity.

We show further that under weak conditions, the most cost-effective way to level the agents' demand or supply curve is by paying those who invest when others do not. Such run insurance is cheap with large noise, and free in the small-noise limit. If run insurance does not suffice to attain the first-best, it is supplemented with small additional payments at a range of states when all agents invest. Finally, under weak assumptions, the cheapest run insurance is floor-based: an agent is guaranteed that if she invests, her payoff will not fall below a given floor as a result of other agents not investing.

Various extensions are also considered: firm-provided insurance, a price cap, taxes on investors, and noise in the policymaker's information. We also show how to extend our techniques to a dynamic model in which participants learn about fundamentals, as well as to the case of duopoly competition.

## 5. Proofs

This section contains the omitted proofs from section 2. The omitted proofs from section 3 can be found in our online appendix (Frankel, 2017).

**Proof of Theorem 2.** The proof relies on an iterative dominance argument that begins in regions where the agents have a strictly dominant action. For any relative payoff function $h$ and price $p$, the lower and upper boundaries of these "dominance regions" are defined, respectively, as follows:

$$\underline{\theta}_h^p = \inf\left\{\theta : h_\theta^0 \le p\right\} \text{ and } \overline{\theta}_h^p = \sup\left\{\theta : h_\theta^1 \ge p\right\}. \tag{27}$$

At any state $\theta$ below the lower boundary $\underline{\theta}_h^p$ (resp., above the upper boundary $\overline{\theta}_h^p$), it is strictly dominant (not) to invest at the price $p$ since, by AM, $h_\theta^\ell > p$ (resp., $h_\theta^\ell < p$) for any investment rate $\ell$. These boundaries are finite for any price $p$:

**Claim 3.** *Assume the relative payoff function $h$ satisfies AM and MSM. Then for any price $p$, the boundaries $\underline{\theta}_h^p$ and $\overline{\theta}_h^p$ are finite.*

**Proof.** For any price $p$ in $\Re$, $\theta' \stackrel{d}{=} \inf\{\theta : H_\theta \le p + k_1\}$ and $\theta'' \stackrel{d}{=} \sup\{\theta : H_\theta \ge p - k_1\}$ are finite by MSM. By AM, for all $\ell$, $h_\theta^\ell \in [H_\theta - k_1, H_\theta + k_1]$ and thus $h_\theta^0 \ge H_\theta - k_1 > p$ for all $\theta < \theta'$ and $h_\theta^1 \le H_\theta + k_1 < p$ for all $\theta > \theta''$. It follows that $\left(\underline{\theta}_h^p, \overline{\theta}_h^p\right) \subset \left(\theta', \theta''\right)$, whence $\underline{\theta}_h^p$ and $\overline{\theta}_h^p$ are also finite. $\square$

Suppose an agent $i$ sees the signal $x$ and believes that each other agent $j$ will use the investment threshold $k$. Under this belief, the law of large numbers implies that the proportion $\ell$

of agents who invest at a given state $\theta$ will be $\Pr\left(\theta+\sigma\varepsilon_j<k|\theta\right)=F\left(\frac{k-\theta}{\sigma}\right)$. Hence, agent $i$'s expected relative payoff $\pi_\sigma(x,k)$ from investing, gross of the price $p$, equals

$$\pi_\sigma(x,k)\overset{d}{=}\int_{\theta=x-\sigma/2}^{x+\sigma/2}\omega_\sigma(\theta|x)\,h_\theta^{F\left(\frac{k-\theta}{\sigma}\right)}d\theta \tag{28}$$

where, by Bayes's Rule,

$$\omega_\sigma(\theta|x)=\frac{\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)\phi(\theta)}{\int_{\theta'=x-\sigma/2}^{x+\sigma/2}\frac{1}{\sigma}f\left(\frac{x-\theta'}{\sigma}\right)\phi(\theta')\,d\theta'} \tag{29}$$

is the agent's posterior density of the state $\theta$ given her signal $x$.[40]

**Lemma 1.** *For any fixed $\sigma>0$, the posterior density $\omega_\sigma(\theta|x)$ defined in (29) is continuous in the signal $x$ and in the state $\theta$.*

**Proof.** Fix an $(x,\theta)\in\Re^2$. We will show that for any $\varepsilon>0$, there is a $\delta\in(0,\sigma/2]$ such that for any $x'$ and $\Delta$ satisfying $\max\left\{\left|x'-x\right|,|\Delta|\right\}\le\delta$, we have $\left|\omega_\sigma\left(\theta+\Delta|x'\right)-\omega_\sigma(\theta|x)\right|\le\varepsilon$. By assumption, $\phi$ has a finite upper bound $\overline{\phi}$. Since $\phi$ is continuous and positive, it also attains a positive minimum $\underline{\phi}$ on the compact set $I=[x-\sigma,x+\sigma]$. Let $\overline{f}=\max_{z\in[-1/2,1/2]}f(z)$. For $x''\in\left\{x,x'\right\}$, $\frac{1}{\sigma}\int_{\theta'=x-\delta-\sigma/2}^{x+\delta+\sigma/2}f\left(\frac{x''-\theta'}{\sigma}\right)d\theta'$ equals one as it is the integral of the posterior density of the state given the signal $x''$ over all states $\theta'$ that are possible given $x''$. Since, moreover, each $\theta'$ in the integral lies in $I$ and thus $\phi\left(\theta'\right)\in\left[\underline{\phi},\overline{\phi}\right]$ it follows that $\int_{\theta'=x-\delta-\sigma/2}^{x+\delta+\sigma/2}f\left(\frac{x''-\theta'}{\sigma}\right)\phi\left(\theta'\right)d\theta'\in\left[\sigma\underline{\phi},\sigma\overline{\phi}\right]$. Since $f$ is continuous, it is uniformly continuous on any compact set by the Heine–Cantor theorem. Moreover, $\phi$ is continuous. Hence, for any $\varepsilon'>0$ there exists a $\delta'>0$ (possibly dependent on $\theta$) such that (a) if $z,z'\in I$ and $\left|z'-z\right|\le\delta'$ then $\left|f\left(z'\right)-f(z)\right|\le\varepsilon'$ and (b) if $|\Delta|<\delta'$ then $|\phi(\theta+\Delta)-\phi(\theta)|<\varepsilon'$. Let $\varepsilon'=\varepsilon\left[\sigma\underline{\phi}\right]^2\left[\left(\overline{\phi}+\overline{f}\right)\left(\sigma\underline{\phi}\right)+\overline{f}\overline{\phi}\right]^{-1}>0$, and let $\delta'$ be the corresponding constant. Let $\delta=\min\left\{\sigma/2,\delta'\sigma/2,\delta'\right\}$, whence $|\Delta|$, $\left|\frac{x'-\theta-\Delta}{\sigma}-\frac{x-\theta}{\sigma}\right|$, and $\left|\frac{x'-\theta'}{\sigma}-\frac{x-\theta'}{\sigma}\right|$ (for all $\theta'$) are all less than $\delta'$ which implies that $|\phi(\theta+\Delta)-\phi(\theta)|$, $\left|f\left(\frac{x'-\Delta-\theta}{\sigma}\right)-f\left(\frac{x-\theta}{\sigma}\right)\right|$, and $\int_{\theta'=x-\delta-\sigma/2}^{x+\delta+\sigma/2}\left|f\left(\frac{x-\theta'}{\sigma}\right)-f\left(\frac{x'-\theta'}{\sigma}\right)\right|\phi\left(\theta'\right)d\theta'$ are all less than $\varepsilon'$. Hence, by the triangle inequality,

$$\begin{aligned}&\left|\omega_\sigma\left(\theta+\Delta|x'\right)-\omega_\sigma(\theta|x)\right|\\ &\le\left[\begin{array}{c}\phi(\theta)\left|f\left(\frac{x'-\theta-\Delta}{\sigma}\right)-f\left(\frac{x-\theta}{\sigma}\right)\right|\\ +f\left(\frac{x'-\theta-\Delta}{\sigma}\right)|\phi(\theta+\Delta)-\phi(\theta)|\end{array}\right]\left[\int_{\theta'=x-\delta-\sigma/2}^{x+\delta+\sigma/2}f\left(\frac{x-\theta'}{\sigma}\right)\phi\left(\theta'\right)d\theta'\right]^{-1}\end{aligned}$$

[40] The posterior density $w_\sigma(\theta|x)$ is given by (29) as long as the denominator in (29) is positive: when $x$ lies in the interior $(-\sigma/2,1+\sigma/2)$ of the set of possible signals. If instead $x$ equals the lowest (highest) possible signal $-\sigma/2$ (resp., $1+\sigma/2$), then the posterior a Dirac delta function with all of its weight on the lowest state $\theta=0$ (resp., the highest state $\theta=1$).

$$+\frac{f\left(\frac{x'-\theta-\Delta}{\sigma}\right)\phi\left(\theta+\Delta\right)\int_{\theta'=x-\delta-\sigma/2}^{x+\delta+\sigma/2}\left|f\left(\frac{x-\theta'}{\sigma}\right)-f\left(\frac{x'-\theta'}{\sigma}\right)\right|\phi\left(\theta'\right)d\theta'}{\left[\int_{\theta'=x-\delta-\sigma/2}^{x+\delta+\sigma/2}f\left(\frac{x'-\theta'}{\sigma}\right)\phi\left(\theta'\right)d\theta'\right]\left[\int_{\theta'=x-\delta-\sigma/2}^{x+\delta+\sigma/2}f\left(\frac{x-\theta'}{\sigma}\right)\phi\left(\theta'\right)d\theta'\right]}$$
$$\leq \frac{\overline{\phi}\varepsilon'+\overline{f}\varepsilon'}{\sigma\underline{\phi}}+\overline{f\phi}\frac{\varepsilon'}{\left[\sigma\underline{\phi}\right]^2}$$

which equals $\varepsilon$. □

**Lemma 2.** *Assume AM.*

1. *For all $x \in \Re$, $\pi_\sigma(x,k)$ is nondecreasing in $k \in \Re$.*
2. *$\pi_\sigma(x,k)$ is continuous in $(x,k) \in \Re^2$.*

**Proof.** Part 1. Fix $x \in \Re$ and let $k' > k$. Then by (28),

$$\pi_\sigma\left(x,k'\right)-\pi_\sigma\left(x,k\right)=\frac{\int_{\theta=-\infty}^{\infty}f\left(\frac{x-\theta}{\sigma}\right)\left[r_\theta^{F\left(\frac{k'-\theta}{\sigma}\right)}-r_\theta^{F\left(\frac{k-\theta}{\sigma}\right)}\right]d\Phi\left(\theta\right)}{\int_{\theta=-\infty}^{\infty}f\left(\frac{x-\theta}{\sigma}\right)d\Phi\left(\theta\right)},$$

whence $\pi_\sigma\left(x,k'\right) \geq \pi_\sigma(x,k)$ by AM: $\pi_\sigma(x,k)$ is nondecreasing in $k$. Part 2. Fix $(x,k) \in \Re^2$. We will show that for any $\varepsilon \in (0,\sigma]$, there is a $\delta \in \left(0,\sigma/2\right]$ such that for any $x'$ and $k'$ satisfying $\max\left\{\left|x'-x\right|,\left|k'-k\right|\right\}<\delta$, we have $\left|\pi_\sigma\left(x',k'\right)-\pi_\sigma(x,k)\right|<\varepsilon$. Let $\Delta$ denote $k'-k$. Doing the change of variables $\theta'=\theta-\Delta$ (whence $k'-\theta=k-\theta'$) and then renaming $\theta'$ to $\theta$, we have $\pi_\sigma\left(x',k'\right)=\int_{\theta=x'-\Delta-\sigma/2}^{x'-\Delta+\sigma/2}\omega_\sigma\left(\theta+\Delta|x'\right)r_\theta^{F\left(\frac{k-\theta}{\sigma}\right)}d\theta$. Thus, by the Cauchy–Schwarz inequality,

$$\begin{aligned}
&\left|\pi_\sigma\left(x',k'\right)-\pi_\sigma(x,k)\right| \\
&=\left|\int_{\theta=x-3\sigma/2}^{x+3\sigma/2}\left[\omega_\sigma\left(\theta+\Delta|x'\right)-\omega_\sigma\left(\theta|x\right)\right]r_\theta^{1-F\left(\frac{k-\theta}{\sigma}\right)}d\theta\right| \\
&\leq\sqrt{\int_{\theta=x-3\sigma/2}^{x+3\sigma/2}\left[\omega_\sigma\left(\theta+\Delta|x'\right)-\omega_\sigma\left(\theta|x\right)\right]^2d\theta}\sqrt{\int_{\theta=x-3\sigma/2}^{x+3\sigma/2}\left[r_\theta^{F\left(\frac{k-\theta}{\sigma}\right)}\right]^2d\theta}.
\end{aligned}\tag{30}$$

The second square root is no greater than $\overline{r}\sqrt{3\sigma}$ where $\overline{r}>0$ is the (finite by assumption) maximum of $\left|r_\theta^\ell\right|$ over pairs $(\theta,\ell)$ in the compact set $\left[x-3\sigma/2,x+3\sigma/2\right]\times[0,1]$. By Lemma 1 and the Heine–Cantor theorem, $\omega_\sigma$ is uniformly continuous in both arguments on any compact set. Thus, there is a $\delta' \in \left(0,\sigma/2\right]$ such that for any $x'$ and $\Delta$ satisfying $\max\left\{\left|x'-x\right|,|\Delta|\right\}<\delta'$, we have $\left|\omega_\sigma\left(\theta+\Delta|x'\right)-\omega_\sigma\left(\theta|x\right)\right|<\varepsilon\left(3\sigma\overline{r}\right)^{-1}$ for all $\theta\in\left[x-3\sigma/2,x+3\sigma/2\right]$ whence the first square root in (30) is less than $\varepsilon\left(3\sigma\overline{r}\right)^{-1}\sqrt{3\sigma}$. Hence, $\left|\pi_\sigma\left(x',k\right)-\pi_\sigma(x,k)\right|\leq\varepsilon$ as claimed. □

The remainder of the proof relies not only on AM, but also on MSM and OSL. Fix a price $p \in S$ and, for any $k$ in $\Re$, define $\overline{\beta}(k)=\sup\{x:\pi_\sigma(x,k)\geq p\}$ and $\underline{\beta}(k)=$

$\inf\{x : \pi_\sigma(x,k) \le p\}$. As noted above, (not) investing is strictly dominant if $x \le \underline{\theta}_r^p - \sigma/2$ (resp., if $x \ge \overline{\theta}_r^p + \sigma/2$) and thus for such $x$, $\pi_\sigma(x,k) > p$ (resp., $\pi_\sigma(x,k) < p$). Hence, both $\overline{\beta}(k)$ and $\underline{\beta}(k)$ are finite by Claim 3. By Lemma 2, each is also nondecreasing in $k$. Moreover, $\pi_\sigma(x,k) < p$ for all $x > \overline{\beta}(k)$, and $\pi_\sigma(x,k) > p$ for all $x < \underline{\beta}(k)$. Hence, by AM, if all others are known (not) to invest when their signals are less (greater) than $k$, then it is optimal for a given agent (not) to invest when her signal is less than $\underline{\beta}(k)$ (greater than $\overline{\beta}(k)$).

Let $\underline{k}_0 = \underline{\theta}_r^p - \sigma/2$ and, for $n = 1, 2, ...,$ let $\underline{k}_n = \underline{\beta}\left(\underline{k}_{n-1}\right)$. For any signal $x$ below $\underline{k}_0$ it is strictly dominant to invest so, in particular, $\pi_\sigma\left(x, \underline{k}_0\right) > p$, whence $\underline{\beta}\left(\underline{k}_0\right) \ge \underline{k}_0$. Since $\underline{\beta}(k)$ is, moreover, nondecreasing in $k$, the sequence $\left(\underline{k}_n\right)_{n=0}^{\infty}$ is nondecreasing by induction. It is bounded above by $\overline{\theta}_r^p + \sigma/2$, so it converges to a limit $\underline{k}$ by the monotone convergence theorem, and all agents invest if their signals are below $\underline{k}$. By part 2 of Lemma 2, $\pi_\sigma\left(\underline{k}_n, \underline{k}_{n-1}\right) = p$ for all $n$ and $\lim_{n\to\infty} \pi_\sigma\left(\underline{k}_n, \underline{k}_{n-1}\right) = \pi_\sigma\left(\underline{k}, \underline{k}\right)$; thus, $\pi_\sigma\left(\underline{k}, \underline{k}\right) = p$. We can construct an analogous sequence $\overline{k}_0 = \overline{\theta}_r^p + \sigma/2$ and, for $n = 1, 2, ...,$ $\overline{k}_n = \overline{\beta}\left(\overline{k}_{n-1}\right)$, which converges to a limit $\overline{k}$ such that no agents invest if their signals exceed $\overline{k}$ (whence $\overline{k} \ge \underline{k}$) and $\pi_\sigma\left(\overline{k}, \overline{k}\right) = p$.

Let $k_\sigma$ denote either $\underline{k}$ or $\overline{k}$. Substituting $\ell = F\left(\frac{k_\sigma - \theta}{\sigma}\right)$,

$$p = \pi_\sigma(k_\sigma, k_\sigma) = \int\limits_{\ell=0}^{1} \alpha(\ell, k_\sigma, \sigma)\, r^\ell_{k_\sigma - \sigma F^{-1}(\ell)} d\ell$$

where $\alpha(\ell, k_\sigma, \sigma) = \frac{\phi\left(k_\sigma - \sigma F^{-1}(\ell)\right)}{\int_{\ell=0}^1 \phi\left(k_\sigma - \sigma F^{-1}(\ell)\right) d\ell}$. To finish the proof, it suffices to show that for all $\varepsilon > 0$, there is a $\delta > 0$ such that for all $\sigma \in (0,\delta)$ and $p$ in $S$, $k_\sigma \in \left(\theta_R^p - \varepsilon, \theta_R^p + \varepsilon\right)$. Suppose otherwise: there is an $\varepsilon > 0$ such that for all $\delta > 0$, there is some $\sigma \in (0,\delta)$ for which $k_\sigma \notin \left(\theta_R^p - \varepsilon, \theta_R^p + \varepsilon\right)$ for some $p$ in $S$. For $n = 1, 2, ...,$ let $\delta_n = 1/n$ and let $\sigma_n \in (0, \delta_n)$ be such that $k_{\sigma_n} \notin \left(\theta_R^p - \varepsilon, \theta_R^p + \varepsilon\right)$. By taking subsequences if needed, we may assume that either $k_{\sigma_n} \le \theta_R^p - \varepsilon$ for all $n$ or $k_{\sigma_n} \ge \theta_R^p + \varepsilon$ for all $n$; w.l.o.g. assume the former. Let $\sigma$ now denote $\sigma_m$ for some $m \ge \lceil 1/\varepsilon \rceil$ that will be specified later. As $\sigma < \varepsilon$, we have $k_\sigma + \sigma/2 < \theta_R^p - \varepsilon/2$. Now, by MSM and (16) we have $R_\theta > p$ for all $\theta < \theta_R^p$ and thus, again by MSM, $\int_{\ell=0}^1 r^\ell_{k_\sigma + \sigma/2} d\ell = R_{k_\sigma + \sigma/2} > R_{\theta_R^p - \varepsilon/2} \ge p + k_3\varepsilon/2 = \pi_\sigma(k_\sigma, k_\sigma) + k_3\varepsilon/2$. Thus, $\frac{k_3\varepsilon}{2} \le \int_{\ell=0}^1 r^\ell_{k_\sigma+\sigma/2} d\ell - \pi_\sigma(k_\sigma, k_\sigma) = A + B$ where $A = \int_{\ell=0}^1 r^\ell_{k_\sigma+\sigma/2}\left[1 - \alpha(\ell, k_\sigma, \sigma)\right] d\ell$ and $B = \int_{\ell=0}^1 \alpha(\ell, k_\sigma, \sigma)\left[r^\ell_{k_\sigma+\sigma/2} - r^\ell_{k_\sigma - \sigma F^{-1}(\ell)}\right] d\ell$. $B$ is a weighted average of terms $r^\ell_{k_\sigma+\sigma/2} - r^\ell_{k_\sigma-\sigma F^{-1}(\ell)}$ each of which, by OSL, is at most $k_5\sigma$; hence, $B \le k_5\sigma$. Thus, $\frac{k_3\varepsilon}{2} - k_5\sigma$ is at most $A$ which cannot exceed $|A|$ which, by the Cauchy–Schwarz inequality, is at most $CD$ where $C = \sqrt{\int_{\ell=0}^1 \left[r^\ell_{k_\sigma+\sigma/2}\right]^2 d\ell}$ and $D = \sqrt{\int_{\ell=0}^1 \left[1 - \alpha(\ell, k_\sigma, \sigma)\right]^2 d\ell}$. As $r$ is bounded on compact sets and $k_\sigma \in \left[\underline{\theta}_r^p, \overline{\theta}_r^p\right]$, $C$ cannot exceed the maximum $\overline{r}$ of $\left|r_\theta^\ell\right|$ over pairs $(\theta, \ell)$ in the compact set $I \times [0,1]$ where $I = \left[\underline{\theta}_r^p - \sigma/2, \overline{\theta}_r^p + \sigma/2\right]$. As for $D$, $\phi(\theta)$ is bounded below by some $\underline{\phi} > 0$ on the compact set $I$. And since $\phi$ is continuous, it is uniformly continuous on any compact set by the Heine–Cantor theorem. Hence, for any $\varepsilon' > 0$ there exists a $\delta' > 0$ such that if $\left|\theta' - \theta''\right| \le \delta'$ and $\theta', \theta'' \in I$ then $\left|\phi\left(\theta'\right) - \phi\left(\theta''\right)\right| \le \varepsilon'$. Select any $\varepsilon'$ in $\left(0, \frac{k_3\underline{\phi}\varepsilon}{4\overline{r}}\right)$ and let $\delta'$ be the corresponding constant. Finally, let the index $m$ be large enough that $\sigma = \sigma_m \leq \min\left\{\varepsilon, \delta', \frac{k_3\varepsilon}{4k_5}\right\}$. Then since $F^{-1}(\ell) \in \left[-1/2, 1/2\right]$, by the triangle inequality,

$$D \leq \max_{\ell \in [0,1]} \left|1 - \alpha\left(\ell, k_\sigma, \sigma\right)\right|$$

$$\leq \max_{\ell \in [0,1]} \frac{\int_{\ell'=0}^{1} \left|\phi\left(k_\sigma - \sigma F^{-1}\left(\ell'\right)\right) - \phi\left(k_\sigma - \sigma F^{-1}\left(\ell\right)\right)\right| d\ell'}{\int_{\ell'=0}^{1} \phi\left(k_\sigma - \sigma F^{-1}\left(\ell'\right)\right) d\ell'} \leq \frac{\varepsilon'}{\underline{\phi}}$$

Thus, $\frac{k_3\varepsilon}{4} < \frac{k_3\varepsilon}{2} - k_5\sigma \leq |A| \leq CD \leq \overline{r}\frac{\varepsilon'}{\underline{\phi}} < \frac{k_3\varepsilon}{4}$, a contradiction. $\square$

**Proof of Theorem 3.** The following lemmas assume $h$ satisfies AM, MSM, and OSL. For any price $p$, let $P_\sigma(p)$ denote the firm's payoff from the price $p$ according to some strategy profile of the agents that survives iterative strict dominance. We first show that this payoff converges to the function

$$P(p) = p\Phi\left(\theta_h^p\right), \tag{31}$$

uniformly in the price $p \in \left[0, \overline{p}\right]$:

**Lemma 3.** *For any $\varepsilon > 0$ there is a $\delta > 0$ such that for all $\sigma \in (0, \delta)$ and every price $p$ in $\left[0, \overline{p}\right]$, $\left|P_\sigma(p) - P(p)\right| < \varepsilon$.*

**Proof.** Let $\overline{\phi} < \infty$ be an upper bound on the prior density $\phi$ of $\theta$. By Theorem 2, for $\varepsilon' = \varepsilon\left(2\overline{p}\overline{\phi}\right)^{-1}$ there exists a $\delta > 0$ such that for any price $p \in \left[0, \overline{p}\right]$ and any private noise scale factor $\sigma$ in the interval $(0, \delta)$, the firm's profit $P_\sigma(p)$ from the price $p$ lies in $\left[\underline{P}(p), \overline{P}(p)\right]$ where $\underline{P}(p) = p\Phi\left(\theta_H^p - \varepsilon'\right)$ and $\overline{P}(p) = p\Phi\left(\theta_H^p + \varepsilon'\right)$. But this interval also contains $P(p)$. Hence $\left|P(p) - P_\sigma(p)\right| \leq \left|\overline{P}(p) - \underline{P}(p)\right| \leq 2\overline{p}\overline{\phi}\varepsilon' = \varepsilon$ as claimed. $\square$

**Lemma 4.** *The function $P(p)$ is continuous in $p \in \left[0, \overline{p}\right]$ and strictly positive for $p \in (0, \overline{p}]$.*

**Proof of Lemma 4.** It is strictly positive since $\Phi$ has full support and $\theta_H^p$ cannot be less than $\overline{\theta}_h^p$ which is finite by Claim 3. Moreover, the products and compositions of continuous functions are continuous. Hence $P$ is continuous by the following lemma.

**Lemma 5.** *$\theta_H^p$ is a Lipschitz-continuous function of $p$ with Lipschitz constant $1/k_3$.*

**Proof.** For any $p'' > p'$, let $\theta' = \theta_H^{p'}$ and $\theta'' = \theta_H^{p''}$. By MSM, $\theta' \geq \theta''$. It remains to show that $\theta' - \theta'' \leq (1/k_3)\left(p'' - p'\right)$. If $\theta' = \theta''$ we are done. If instead $\theta' > \theta''$ then, for any $\varepsilon > 0$, (a) $H_{\theta'-\varepsilon} \geq p'$ as $\theta' = \inf\left\{\theta : H_\theta < p'\right\}$ and (b) $H_{\theta''+\varepsilon} \leq p''$ as $\theta'' = \sup\left\{\theta : H_\theta > p''\right\}$. Hence, $p' - p'' \leq H_{\theta'-\varepsilon} - H_{\theta''+\varepsilon}$. If, in addition, $\varepsilon < \frac{\theta'-\theta''}{2}$, then $\theta' - \varepsilon > \theta'' + \varepsilon$ so by MSM, $H_{\theta'-\varepsilon} - H_{\theta''+\varepsilon} \leq -k_3\left(\theta' - \theta'' - 2\varepsilon\right)$. Combining equalities we obtain $p' - p'' \leq -k_3\left(\theta' - \theta'' - 2\varepsilon\right)$. Since this holds for all $\varepsilon > 0$, it must also hold in the limit as $\varepsilon \to 0$ as $p'$, $p''$, $\theta'$, and $\theta''$ do not depend on $\varepsilon$: $\theta' - \theta'' \leq (1/k_3)\left(p'' - p'\right)$ as claimed. $\square$

If $P$ has a unique maximizer $p_0$, then the firm's optimal price must converge to $p_0$:

**Lemma 6.** *For each noise scale factor $\sigma$, let the price $p_\sigma$ be a best response for the firm to some strategy profile of the agents that survives iterative strict dominance. If $P$ has a unique maximizer $p_0 \in (0, \infty)$, then $\lim_{\sigma \to 0} p_\sigma = p_0$.*

**Proof.** Suppose not: there is an $\varepsilon > 0$ such that for all $n = 1, 2, ...$ there is a $\sigma_n \in (0, 1/n)$ such that $p_{\sigma_n}$ lies in the set $I = [0, p_0 - \varepsilon] \cup [p_0 + \varepsilon, \overline{p}]$. Since $I$ is compact, by restricting to subsequences if needed we may assume that $(p_{\sigma_n})_{n=1}^{\infty}$ converges to a limit $p^*$ in $I$. Recall that $P_\sigma(p)$ denotes the firm's profit from the price $p$ when the scale factor is $\sigma$. Now, $P_{\sigma_n}(p_0) - P_{\sigma_n}(p_{\sigma_n}) = a_n + b + c_n + d_n$ where $a_n = P_{\sigma_n}(p_0) - P(p_0)$, $b = P(p_0) - P(p^*) > 0$, $c_n = P(p^*) - P(p_{\sigma_n})$, and $d_n = P(p_{\sigma_n}) - P_{\sigma_n}(p_{\sigma_n})$. By Lemma 3 and Lemma 4, there is an $n^*$ such that for all $n > n^*$, $|a_n|$, $|c_n|$, and $|d_n|$ are all at most $b/4$. It follows that for such $n$, $P_{\sigma_n}(p_0) - P_{\sigma_n}(p_{\sigma_n}) \geq b/4 > 0$, which contradicts the optimality of $p_{\sigma_n}$ when the noise scale factor is $\sigma_n$. $\square$

**Lemma 7.**

1. *For any price $p$, $P(p) \leq P\left(H_{\theta_h^p}\right)$.*
2. *If the price $p$ maximizes $P$, then $p$ equals $H_\theta$ where $\theta$ equals $\theta_H^p$ and maximizes $\Pi_h$. Conversely, if the investment threshold $\theta$ maximizes $\Pi_h$, then the price $p = H_\theta$ maximizes $P$.*

**Proof.** Part 1. Fix a price $p$ and let $\theta = \theta_H^p$ be the resulting investment threshold. Now fix this state $\theta$ and redefine $p$ to be any price in the interval $[H_\theta^+, H_\theta]$. (If $H$ is continuous at $\theta$, this interval consists of a single point.) By MSM and (16), $\theta_H^p$ equals the fixed state $\theta$ for all such prices $p$. So by (31), the firm's payoff $P(p)$ is strictly increasing in $p \in [H_\theta^+, H_\theta]$, whence $P(p) \leq P\left(H_{\theta_h^p}\right)$ as claimed. Part 2. Continuing the example of part 1, if the price $p$ that maximizes $P$ lies in $[H_\theta^+, H_\theta]$, it must equal $H_\theta$. Any price $p$ that maximizes $P$ must then be of the form $H_\theta$ where $\theta$ equals $\theta_H^p$. In this case, $\theta$ must also maximize $\Pi_h$. For suppose not. Then there is a $\theta'$ such that $\Pi_h(\theta') > \Pi_h(\theta)$. Let $p'$ be the price $H_{\theta'}$, which satisfies $\theta_H^{p'} = \theta'$. Thus, $P(p') = \Pi_h(\theta') > \Pi_h(\theta) = P(p)$, a contradiction. Now suppose that $\theta$ maximizes $\Pi_h$, but the price $p = H_\theta$ does not maximize $P$: there is another price $p'$ that does. By the preceding reasoning, $p'$ must be of the form $H_{\theta'}$ where $\theta' = \theta_H^{p'}$, whence $\Pi_h(\theta') = P(p') > P(p) = \Pi_h(\theta)$, a contradiction. $\square$

We can now prove parts 1 and 2 of Theorem 3.

1. Since $\widehat{\theta}$ uniquely maximizes $\Pi_h$, the price $p_0 = H_{\widehat{\theta}}$ uniquely maximizes $P$ by part 2 of Lemma 7. Thus, by Lemma 6, $\lim_{\sigma \to 0} p_\sigma = p_0$, so by Lemma 5, $\lim_{\sigma \to 0} \theta_H^{p_\sigma}$ equals $\theta_H^{p_0}$ which, by MSM, equals $\widehat{\theta}$.
2. Since $\widehat{\theta}$ is not a weak maximizer of $\Pi_h$, there is another state $\theta'$ such that $\Pi_h(\theta') > \Pi_h(\widehat{\theta})$. Let $p' = H_{\theta'}$ and $p_m = H_{\widehat{\theta}}$. By MSM and (16), $\theta_H^{p'} = \theta'$ and $\theta_H^{p_m} = \widehat{\theta}$. By (17) and (31), $0 < k \stackrel{d}{=} \Pi_h(\theta') - \Pi_h(\widehat{\theta}) = P(p') - P(p_m)$. By (17) and MSM, $\Pi_h(\theta)$ is left-continuous in $\theta$. By MSM, $H$ is decreasing so $\Pi_h(\theta)$ cannot jump upwards as $\theta$ rises. These two properties of $\Pi_h$ imply that there is a $\delta > 0$ such that

$$\Pi_h(\theta) \leq \Pi_h(\widehat{\theta}) + k/2 \text{ for all } \theta \text{ in } [\widehat{\theta} - \delta, \widehat{\theta} + \delta]. \tag{32}$$

As $p_\sigma$ is optimal when the noise scale factor is $\sigma$ and since $P\left(p'\right) - P\left(p_m\right) = k$,

$$0 \geq P_\sigma\left(p'\right) - P_\sigma\left(p_\sigma\right) = k + a_\sigma + \left[P\left(p_m\right) - P\left(p_\sigma\right)\right] + b_\sigma,$$

where, by Lemma 6, $a_\sigma = P_\sigma\left(p'\right) - P\left(p'\right) \to 0$ and $b_\sigma = P\left(p_\sigma\right) - P_\sigma\left(p_\sigma\right) \to 0$. Hence for any $\varepsilon > 0$ there is a $\sigma^* > 0$ such that for all $\sigma \in (0, \sigma^*)$, $P\left(p_\sigma\right) - P\left(p_m\right) > k/2$ and thus $\Pi_h\left(\theta_H^{p_\sigma}\right) - \Pi_h\left(\widehat{\theta}\right) > k/2$ as $P\left(p_\sigma\right) \leq \Pi_h\left(\theta_H^{p_\sigma}\right)$ by part 1 of Lemma 7 and since $P\left(p_m\right) = \Pi_h\left(\widehat{\theta}\right)$, whence $\left|\theta_H^{p_\sigma} - \widehat{\theta}\right| > \delta$ by (32): $\theta_H^{p_\sigma}$ does not converge to $\widehat{\theta}$. $\square$

**Proof of Theorem 1.** We start with part 2: $p_m \geq r^1_{\theta^*}$. We first prove that no efficient APSS has a cost below $C\left(p_m\right)$. In an efficient APSS, all agents invest if $\theta < \theta^*$ and none do so if $\theta > \theta^*$. Hence the asymptotic cost of an efficient APSS is $\Psi\left(\tau\right) = \int_{\theta \leq \theta^*} \tau^1_\theta d\Phi\left(\theta\right)$. Moreover, augmented demand $\widetilde{R}$ in an efficient APSS cannot lie below the function

$$\widehat{R}_\theta = \begin{cases} R_\theta & \text{if} \quad \theta > \theta^* \\ \max\left\{R_\theta, p_m\right\} & \text{if} \quad \theta \leq \theta^* \end{cases}$$

by (13) and since augmented demand $\widetilde{R}$ is decreasing by MSM and cannot lie below primitive demand $R$. Moreover, for any efficient APSS and any $\theta < \theta^*$ we must have $\widetilde{r}^1_\theta \geq \widetilde{R}_\theta \geq \widetilde{R}_{\theta^*} \geq p_m$ by AM($\widetilde{r}$), MSM, and (13), respectively, and thus $\tau^1_\theta = \widetilde{r}^1_\theta - r^1_\theta \geq p_m - r^1_\theta$. Since, in addition, $\tau^1_\theta$ is nonnegative, the asymptotic cost of any such scheme $\Psi\left(\tau\right)$ is at least $C\left(p_m\right)$ as claimed.

Fix $\varepsilon > 0$. We now produce an efficient APSS $\tau$ with a cost $\Psi\left(\tau\right) < C\left(p_m\right) + \varepsilon$. Fix constants $\iota > 0$ and $k'_3 \in (0, k_3)$ to be determined later. Let $\theta_1$ and $\theta_2$ be given by (18) and (19), and let $\tau$ be given by (21). Let $\widetilde{r}$ denote the augmented relative payoff function $r + \tau$.

**Lemma 8.** *Assume AM($r$), SM($r$), and PMC($r$). Then AM($\widetilde{r}$), MSM($\widetilde{r}$), and OSL($\widetilde{r}$) hold for all $\theta$.*

**Proof.** By construction $\widetilde{R}$ is left-continuous at $\theta^*$ where it jumps downwards; falls at a rate between $k_3$ and $k_4$ at states $\theta \leq \theta_1$ and $\theta > \theta^*$; and falls at the rate $k'_3 \in (0, k_3)$ at states in $(\theta_1, \theta^*)$. Hence, MSM($\widetilde{r}$) holds with $k_3$ replaced by $k'_3$. AM and OSL are proved for different intervals of $\theta$ as follows.

1. The intervals $(-\infty, \theta_1]$ and $(\theta^*, \infty)$. Here $\widetilde{r}$ coincides with $r$ and thus satisfies AM and OSL by Claim 1.
2. The interval $\left[\theta_2, \theta^*\right]$. Here $\widetilde{r}^\ell_\theta$ equals $\widetilde{R}_\theta = p_m + \iota + k'_3\left(\theta^* - \theta\right)$. It satisfies AM as it is independent of $\ell$. It satisfies OSL since for any $\ell$ and $\theta' > \theta$ in $\left[\theta_2, \theta^*\right]$, $\widetilde{r}^\ell_{\theta'} - \widetilde{r}^\ell_\theta$ equals $\widetilde{R}_{\theta'} - \widetilde{R}_\theta$ which is negative.
3. The interval $[\theta_1, \theta_2]$. By (21), $\widetilde{r}^\ell_\theta = \left(1 - \alpha_\theta\right) r^\ell_\theta + \alpha_\theta r^1_\theta$. Hence, for $\ell' > \ell$, both $\widetilde{r}^{\ell'}_\theta - \widetilde{r}^\ell_\theta = \left(1 - \alpha_\theta\right)\left(r^{\ell'}_\theta - r^\ell_\theta\right)$ and $\widetilde{r}^\ell_\theta - \widetilde{r}^0_\theta = \left(1 - \alpha_\theta\right)\left(r^\ell_\theta - r^0_\theta\right) + \alpha_\theta\left(r^1_\theta - r^0_\theta\right)$ are nonnegative and lie in $[0, k_1]$ by AM($r$) and since $\alpha_\theta \in [0, 1]$: AM($\widetilde{r}$) holds. As for OSL($\widetilde{r}$), let $\theta' > \theta$ both lie in $[\theta_1, \theta_2]$. By SM($r$) and AM($r$), $T_\theta \leq T_{\theta^*} = r^1_{\theta^*} - R_{\theta^*} \leq k_1$; by SM($r$), $\left|T_{\theta'} - T_\theta\right| \leq \left(k'_3 + k_4\right)\left|\theta' - \theta\right|$. Thus, by SM($r$), PMC($r$) and the triangle inequality,

$$\left|\alpha_{\theta'} - \alpha_\theta\right| \leq \left|\frac{T_{\theta'} - T_\theta}{r^1_{\theta'} - R_{\theta'}}\right| + T_\theta\left|\frac{r^1_\theta - R_\theta - \left(r^1_{\theta'} - R_{\theta'}\right)}{\left(r^1_{\theta'} - R_{\theta'}\right)\left(r^1_\theta - R_\theta\right)}\right|$$

$$\leq \left( \frac{k_3' + k_4}{k_2} + k_1 \frac{k_4 - k_3}{k_2^2} \right) (\theta' - \theta)$$

whence by SM$(r)$ and the triangle inequality,

$$\left| \widetilde{r}_{\theta'}^{\ell} - \widetilde{r}_{\theta}^{\ell} \right| \leq |\alpha_{\theta'} - \alpha_\theta| \left( r_{\theta'}^1 - r_{\theta'}^{\ell} \right) + \alpha_\theta \left| \left( r_{\theta'}^1 - r_{\theta'}^{\ell} \right) - \left( r_\theta^1 - r_\theta^{\ell} \right) \right| + \left| r_{\theta'}^{\ell} - r_\theta^{\ell} \right|$$

which is at most $k_5 \left( \theta' - \theta \right)$ where $k_5 = \frac{k_3'+k_4}{k_2} k_1 + (k_4 - k_3) \left( \frac{k_1^2}{k_2^2} + 1 \right) + k_4$: OSL$(\widetilde{r})$ holds. □

It follows from Lemma 8 that $\tau$ is an APSS. Thus, Theorems 2 and 3 remain valid in the presence of $\tau$, where $R$ in those results is replaced by $\widetilde{R}_\theta = R_\theta + T_\theta$. Hence, by Theorem 3, in the limit as $\sigma$ goes to zero the firm chooses an investment threshold $\theta$ that maximizes the payoff function $\Pi_{\widetilde{r}}(\theta) = \widetilde{R}_\theta \Phi(\theta)$ if there is a unique such maximizer.

It remains to show that the firm's payoff $\Pi_{\widetilde{r}}(\theta^*) = (p_m + \iota) \Phi(\theta^*)$ from $\theta^*$ exceeds its payoff $\Pi_{\widetilde{r}}(\theta)$ from any other threshold $\theta$. For $\theta \leq \theta_1$ or $\theta > \theta^*$ this holds since $\widetilde{R}_\theta = R_\theta$ at such states $\theta$ and since $p_m \Phi(\theta^*) = \max_\theta [R_\theta \Phi(\theta)]$ by (14). Now consider $\theta$ in $[\theta_1, \theta^*]$. Let $\underline{\phi} > 0$ be a lower bound for $\phi(\theta)$ on $\theta \in [\theta_1, \theta^*]$, whence $\Phi(\theta^*) - \Phi(\theta) \geq (\theta^* - \theta) \underline{\phi}$ for all such $\theta$. Since, in addition, $\Phi(\theta) \leq 1$, a sufficient condition for

$$\begin{aligned} 0 < \Pi_{\widetilde{r}}\left(\theta^*\right) - \Pi_{\widetilde{r}}(\theta) &= (p_m + \iota) \Phi\left(\theta^*\right) - \left[p_m + \iota + k_3'\left(\theta^* - \theta\right)\right] \Phi(\theta) \\ &= (p_m + \iota)\left[\Phi\left(\theta^*\right) - \Phi(\theta)\right] - k_3'\left(\theta^* - \theta\right) \Phi(\theta) \end{aligned}$$

to hold for all $\theta$ in $[\theta_1, \theta^*]$ is that $k_3'$ not exceed $\underline{\phi}(p_m + \iota)$ which is positive as $p_m \geq r_{\theta^*}^1 > s_{\theta^*} = 0$ by hypothesis, (10), and DMSB, respectively. Hence, $\theta^*$ uniquely maximizes $\Pi_{\widetilde{r}}$ as long as

$$k_3' \in \left(0, \min\left\{k_3, (p_m + \iota)\underline{\phi}\right\}\right). \tag{33}$$

It remains only to show that the asymptotic cost $\Psi(\tau)$ of the scheme converges to $C(p_m)$ as $k_3'$ and then $\iota$ goes to zero. The cost of the scheme is zero at a given state $\theta$ if $\widetilde{R}_\theta \leq r_\theta^1$ as then $\tau_\theta^1 = 0$. And when $\widetilde{R}_\theta > r_\theta^1$, the cost is just $\tau_\theta^1 = \widetilde{R}_\theta - r_\theta^1$. Thus, by (20), the cost can be written as $\int_{\theta \leq \theta^*} \max\left\{p_m + \iota + k_3'(\theta^* - \theta) - r_\theta^1, 0\right\} d\theta$ which is continuous and increasing in $\iota \geq 0$ and $k_3' \geq 0$. Hence, as $\iota$ and $k_3'$ go to zero, the cost converges to $C(p_m)$ as claimed. This proves part 2.

Part 1: assume $p_m < r_{\theta^*}^1$. Let $\iota = r_{\theta^*}^1 - p_m$ and let $k_3'$ be chosen from the interval in (33). Let the scheme $\tau$ again be given by (21); the state $\theta_1$ is given by (19); the state $\theta_2$ given by (18) now coincides with $\theta^*$ by the value chosen for $\iota$. Asymptotic Predictability now follows from parts 1 and 3 of Lemma 8: $\tau$ is an APSS. Accordingly, Theorems 2 and 3 remain valid in the presence of our insurance scheme, where $R$ in that result is replaced by the augmented mean relative payoff function $\widetilde{R}_\theta = R_\theta + T_\theta$. We must show that the firm's augmented limiting payoff function $\Pi_{\widetilde{r}}(\theta) = \widetilde{R}_\theta^- \Phi(\theta)$ is uniquely maximized at the socially optimal threshold $\theta^*$. If so then, by Theorem 3, the APSS must induce the firm to choose an investment threshold that converges to $\theta^*$ in the limit as $\sigma$ vanishes.

Since $\Pi_{\widetilde{r}}(\theta^*)$ equals $r_{\theta^*}^1 \Phi(\theta^*)$, it follows that $\Pi_{\widetilde{r}}(\theta^*) > \Pi_r\left(\widehat{\theta}\right) \geq \Pi_r(\theta) = \Pi_{\widetilde{r}}(\theta)$ for all $\theta$ in the set $\Re \backslash \left(\theta_1, \theta^*\right]$ of states at which no transfers are given, as $\widehat{\theta}$ strictly maximizes $\Pi_r$. Moreover, for $\theta$ in $(\theta_1, \theta^*)$,

$$\Pi_{\widetilde{r}}\left(\theta^*\right)-\Pi_{\widetilde{r}}\left(\theta\right)=r^1_{\theta^*}\Phi\left(\theta^*\right)-\left[r^1_{\theta^*}+k_3'\left(\theta^*-\theta\right)\right]\Phi\left(\theta\right)$$
$$=r^1_{\theta^*}\left[\Phi\left(\theta^*\right)-\Phi\left(\theta\right)\right]-k_3'\left(\theta^*-\theta\right)\Phi\left(\theta\right). \tag{34}$$

Since $\phi\left(\right)>0$, there is a $\underline{\phi}>0$ such that $\phi\left(\theta\right)\geq\underline{\phi}$ for all $\theta$ in $\left[\theta_1,\theta^*\right]$, whence $\Phi\left(\theta^*\right)-\Phi\left(\theta\right)\geq\left(\theta^*-\theta\right)\underline{\phi}$ for all $\theta$ in $(\theta_1,\theta^*)$. Thus, since $\Phi\left(\theta^*\right)>\Phi\left(\theta\right)$, $\Pi_{\widetilde{r}}\left(\theta^*\right)-\Pi_{\widetilde{r}}\left(\theta\right)>r^1_{\theta^*}\left(\theta^*-\theta\right)\underline{\phi}-k_3'\left(\theta^*-\theta\right)\Phi\left(\theta^*\right)$ by (34). Since, moreover, $\Phi\left(\theta^*\right)\leq 1$, in order for $\Pi_{\widetilde{r}}\left(\theta^*\right)$ to exceed $\Pi_{\widetilde{r}}\left(\theta\right)$ for all $\theta$ in $(\theta_1,\theta^*)$ it suffices that $k_3'$ not exceed $\underline{\phi}r^1_{\theta^*}$ which, in turn, lies in $(0,\infty)$ by (3) and since $s_{\theta^*}=0$ by DMSB. Thus, by (33), the limiting payoff function $\Pi_{\widetilde{r}}$ is uniquely maximized at the socially optimal threshold $\theta^*$ and hence, by Theorem 3, induces the firm to choose a threshold that converges to $\theta^*$ in the limit as $\sigma\to 0$. Finally, since $\theta_2$ coincides with $\theta^*$, $\tau^1_\theta$ is identically zero: the scheme is asymptotically costless as claimed. This proves part 1. □

## Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2017.04.001.

## References


Andrade, Gregor, Kaplan, Steven N., 1998. How costly is financial (not economic) distress? Evidence from highly leveraged transactions that became distressed. J. Finance 53, 1443–1493.

Angeletos, George-Marios, Hellwig, Christian, Pavan, Alessandro, 2006. Signalling in a global game: coordination and policy traps. J. Polit. Econ. 114, 452–484.

Angeletos, George-Marios, Pavan, Alessandro, 2009. Policy with dispersed information. J. Eur. Econ. Assoc. 7, 11–60.

Angeletos, George-Marios, Pavan, Alessandro, 2013. Selection-free predictions in global games with endogenous information and multiple equilibria. Theor. Econ. 8, 883–938.

Angeletos, George-Marios, Werning, Ivan, 2006. Crises and prices: information aggregation, multiplicity, and volatility. Am. Econ. Rev. 96, 1720–1736.

Argenziano, Rossella, 2008. Differentiated networks: equilibrium and efficiency. Rand J. Econ. 39, 747–769.

Babina, Tania, 2015. Destructive creation at work: how financial distress spurs entrepreneurship. Finance Dept., UNC-Chapel Hill. Mimeo.

Brown, Jennifer, Matsa, David A., 2016. Boarding a sinking ship? An investigation of job applications to distressed firms. J. Finance 71, 507–550.

Burdzy, Krzysztof, Frankel, David M., 2005. Shocks and business cycles. Adv. Theor. Econ. 5 (1), paper No. 2.

Burdzy, K., Frankel, D.M., Pauzner, A., 2001. Fast equilibrium selection by rational players living in a changing world. Econometrica 69, 163–189.

Carlsson, Hans, van Damme, Eric, 1993. Global games and equilibrium selection. Econometrica 61, 989–1018.

Chamley, Christophe, 1999. Coordinating regime switches. Q. J. Econ. 114, 869–905.

Chassang, Sylvain, 2010. Fear of miscoordination and the robustness of cooperation in dynamic global games with exit. Econometrica 78, 973–1006.

Farrell, Joseph, Katz, Michael, Fall/Winter 1998. The effects of antitrust and intellectual property law on compatibility and innovation. In: Antitrust Bulletin, pp. 609–650.

Frankel, David M., 2012. Recurrent crises in global games. J. Math. Econ. 48, 309–321.

Frankel, David M., 2017. Online appendix to 'Efficient ex-ante stabilization of firms'. Mimeo.

Frankel, David M., Morris, Stephen, Pauzner, Ady, 2003. Equilibrium selection in global games with strategic complementarities. J. Econ. Theory 108, 1–44.

Frankel, David M., Pauzner, Ady, 2000. Resolving indeterminacy in dynamic settings: the role of shocks. Q. J. Econ. 115, 283–304.

Frankel, David M., Pauzner, Ady, 2002. Expectations and the timing of neighborhood change. J. Urban Econ. 51, 295–314.

Goldstein, Itay, Pauzner, Ady, 2004. Contagion of self-fulfilling financial crises due to diversification of investment portfolios. J. Econ. Theory 119, 151–183.

Goldstein, Itay, Pauzner, Ady, 2005. Demand-deposit contracts and the probability of bank runs. J. Finance 60, 1293–1327.
Graham, John R., Kim, Hyunseob, Li, Si, Qiu, Jiaping, 2016. Employee costs of corporate bankruptcy. Available at SSRN: http://ssrn.com/abstract=2276753 or http://dx.doi.org/10.2139/ssrn.2276753.
Greenberg, Alan C., 2010. The Rise and Fall of Bear Stearns. Simon & Schuster, New York.
Heinemann, Frank, Nagel, Rosemarie, Ockenfels, Peter, 2009. Measuring strategic uncertainty in coordination games. Rev. Econ. Stud. 76, 181–221.
Hellwig, Christian, Mukherji, Arijit, Tsyvinski, Aleh, 2006. Self-fulfilling currency crises: the role of interest rates. Am. Econ. Rev. 96, 1769–1787.
Hortaçsu, Ali, Matvos, Gregor, Syverson, Chad, Venkataraman, Sriram, 2012. Indirect costs of financial distress in durable goods industries: the case of auto manufacturers. Rev. Financ. Stud. 26, 1248–1290.
Jullien, Bruno, Pavan, Alessandro, 2016. Platform pricing under dispersed information. Mimeo.
Kim, Youngse, 1996. Equilibrium selection in $n$-person coordination games. Games Econ. Behav. 15, 203–227.
Longstaff, Francis A., 2010. The subprime credit crisis and contagion in financial markets. J. Financ. Econ. 97, 436–450.
Maksimovic, Vojislav, Titman, Sheridan, 1991. Financial policy and reputation for product quality. Rev. Financ. Stud. 4, 175–200.
Morris, Stephen, Shin, Hyun Song, 1998. Unique equilibrium in a model of self-fulfilling currency attacks. Am. Econ. Rev. 88, 587–597.
Morris, Stephen, Shin, Hyun Song, 2000b. Global games: theory and applications. In: Dewatripont, M., Hansen, L., Turnoksky, S. (Eds.), Advances in Economic Theory and Econometrics: Proceedings of the Eighth World Congress of the Econometric Society. Cambridge University Press.
Morris, Stephen, Shin, Hyun Song, 2004a. Liquidity black holes. Rev. Finance 8, 1–18.
Morris, Stephen, Shin, Hyun Song, 2004b. Coordination risk and the price of debt. Eur. Econ. Rev. 48, 133–153.
Morris, Stephen, Shin, Hyun Song, 2006. Heterogeneity and uniqueness in interaction games. In: Blume, Lawrence E., Durlauf, Steven N. (Eds.), The Economy as an Evolving Complex System, III: Current Perspectives and Future Directions. Oxford University Press, United Kingdom.
Oyama, Daisuke, 2004. Booms and slumps in a game of sequential investment with the changing fundamentals. Jpn. Econ. Rev. 55, 311–320.
Titman, Sheridan, 1984. The effect of capital structure on a firm's liquidation decision. J. Financ. Econ. 13, 137–151.
Titman, Sheridan, Wessels, Roberto, 1988. The determinants of capital structure choice. J. Finance 43, 1–19.
Toxvaerd, Flavio, 2008. Strategic merger waves: a theory of musical chairs. J. Econ. Theory 140, 1–26.
U.S. Government Accountability Office, 2009. Auto industry: summary of government efforts and automakers' restructuring to date. Downloaded on 11/8/2011 from www.gao.gov/new.items/d09553.pdf.
Weinstein, Jonathan, Yildiz, Muhamet, 2007. A structure theorem for rationalizability with application to robust prediction of refinements. Econometrica 75, 365–400.
Weyl, E. Glen, 2010. A price theory of multi-sided platforms. Am. Econ. Rev. 100, 1642–1762.

## Further reading

Angeletos, George-Marios, Hellwig, Christian, Pavan, Alessandro, 2007. Dynamic global games of regime change: learning, multiplicity, and the timing of attacks. Econometrica 75, 711–756.
Mathevet, Laurent, Steiner, Jakub, 2013. Tractable dynamic global games and applications. J. Econ. Theory 148, 2583–2619.
Morris, Stephen, Shin, Hyun Song, 2000a. Rethinking multiple equilibria in macroeconomic modelling. In: Bernanke, B., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2000. MIT Press, Cambridge, Mass.