Securitization and Lending Competition: Online Appendix*

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1 Introduction

This document is the online appendix to Frankel and Jin [5], which we first summarize. There are two banks, one local and one remote. There is a continuum of loan applicants who have different credit qualities, which are known only to the local bank. Finally, there is a continuum of risk-neutral investors. The remote bank first makes loan offers, followed by the local bank. Applicants then choose which, if any, offer to accept. The local bank then observes a macroeconomic shock that affects loan repayment probabilities. Finally, each bank chooses a proportion of their loans to offer for sale.

While the local bank has an informational advantage at the lending stage, the macroeconomic shock creates a lemons problem that forces it to retain some of its loans in order to signal their quality. As the remote bank faces no such limitation, its profits from securitizing a given loan portfolio are greater. Hence, remote lending can occur.

In Frankel and Jin [5], investors can infer the distribution of credit qualities in each bank's loan portfolio by observing the measure of loans that each bank makes. Hence, if there were no subsequent macroeconomic shock, there would be no lemons problem for the local bank. Given its informational advantage, all lending would be local.

In this appendix we consider a different approach to creating a lemons problem: aggregate uncertainty about the distribution of credit qualities. We study the simplest case, in which there is a single applicant whose credit quality is unknown.

We first show that there are multiple separating equilibria. In some the remote bank lends while in others it does not. Intuitively, with a single agent investors cannot infer the local bank's lending standards with certainty. Hence, if the local bank deviates (e.g., by selling a higher than expected proportion of its loan), investors may conclude that the borrower has no possibility of repayment. Threatened with such pessimistic beliefs, the local bank can be prevented from securitizing more than an arbitrary proportion of its loans. This permits multiple equilibria with different such arbitrary proportions. When this proportion is low, the remote bank's easier access to the securitization market outweighs the informational disadvantage it faces at the lending stage: remote lending can occur.

When faced with multiple equilibria, researchers often look to equilibrium refinements to sharpen their predictions. In games with a continuum of signals and actions such as ours, one needs a strong selection criterion such as the D1 refinement of Banks and Sobel [2]. We apply D1 and show that a unique equilibrium survives, in which all lending is local. Intuitively, if the local bank sells an unexpectedly high proportion of its loan, D1 forces investors to believe that the loan is of the lowest quality θ_1 to which the local bank lends. (A loan's quality is just its repayment probability.) Hence, the local bank can securitize its entire loan to an applicant of quality θ that is slightly below θ_1 and receive the price $r\theta_1$, where r is the interest rate. For the local bank not to want to deviate in this way, $r\theta_1$ must equal the common cost of capital. If the local bank competes, it attracts the applicant only if her quality is below θ_1. Hence, the most investors will pay for the remote bank's loan is $rE\left[\theta|\theta<\theta_1\right]$, which is less than the cost of capital $r\theta_1$: the remote bank will not lend.

A problem with this result is that the restriction on off-equilibrium beliefs that D1 imposes is somewhat arbitrary. Roughly speaking, D1 states that following a deviation, investors believe with certainty that the loan is of the type θ for which the local bank most benefits from the deviation. However, the local bank may benefit from deviating for other loan types as well. D1 prevents investors from assigning any positive probability to these other loan types. A discussion appears in section 4.1.

2 The Model

There is a single agent with a project that succeeds with probability θ. The random variable θ has the distribution function $G_{\overline{\theta}}$, which is indexed by the mean value $\overline{\theta}=E\left[\theta\right]$ as in Frankel and Jin [5]. The agent's project has a gross return of $\rho>1$ if it succeeds and zero if it fails. It requires a unit of capital that the agent must borrow. There is a local bank and a remote bank. The local bank knows the agent's success probability θ while the remote bank knows only its distribution $G_{\overline{\theta}}$.

The remote bank first chooses whether to make an offer and, if so, at which interest rate

r. Seeing r, the local bank then decides whether or not to make an offer of its own. The applicant then chooses which, if any, offer to accept. (The applicant can accept at most one offer.) Let r equal the gross return ρ if the remote bank refrained from making an offer. With this convention, the agent is willing to pay the local bank the interest rate r but no more. Hence, if the local bank does make an offer, it will offer r and the agent will accept.

Assume an offer is accepted. With probability $a \in (0,1)$, the lending bank can then offer any proportion of its loan for sale to a continuum of uninformed, risk-neutral investors with deep pockets. With complementary probability $1-a$, the bank must hold the loan to maturity.[1] The bank's discount factor is δ while the investors have a unit discount factor. In order to obtain an analytic solution for the interest rate, we focus on the case in which the distribution of types θ is not too concave, and its concavity is nondecreasing in θ:

No Cream Skimming (NCS). For all θ in the interior of the support of $G_{\overline{\theta}}$, $\theta G''_{\overline{\theta}}(\theta)/G'_{\overline{\theta}}(\theta)$ is greater than -1 and is weakly increasing in θ.

We first establish a monotonicity property that must hold in any pure strategy equilibrium. Fix an interest rate r offered by the remote bank. The local bank's payoff from lending without securitization is the discounted expected gross loan return $\delta\theta r$ less the unitary cost of capital. If, in addition, the securitization market functions (probability a) and the bank sells a proportion Q at the price P, the bank gets an additional payoff of $Q[P-\delta\theta r]$. Hence, the local bank's total payoff from lending is

$$U^r(\theta, Q, P) = \delta\theta r - 1 + aQ[P-\delta\theta r] = -1 + \delta\theta r[1-aQ] + aQP \tag{1}$$

which is increasing in the repayment probability θ. Let $P^r(Q)$ be the equilibrium price function. For all $\theta \in [0,1]$, let

$$Q^r(\theta) = \arg\max_{Q\in[0,1]} U^r(\theta, Q, P^r(Q)) \tag{2}$$

[1] In Frankel and Jin [5] there is no constant a as we focus on the limit as $a \to 1$. In this appendix, we must fix $a < 1$ in order to apply D1. As the results do not place conditions on a other than $a < 1$, they also hold in the limit as $a \to 1$.

be the local bank's optimal quantity and let

$$\Pi(\theta) = \max_{Q \in [0,1]} U^r(\theta, Q, P^r(Q)) \tag{3}$$

be its maximum expected profits if it lends to an applicant of type θ. Equations (2) and (3) actually define Q^r and Π on the whole real line, although for $\theta \notin [0,1]$ they have no economic interpretation.

Proposition 1 *Fix $r > 0$. The function Q^r is nonincreasing. The function Π is continuous and increasing and satisfies $\lim_{\theta\to\infty} \Pi(\theta) = \infty$ and $\lim_{\theta\to-\infty} \Pi(\theta) = -\infty$.*

An immediate consequence is that the local bank follows a threshold lending rule:[2]

Corollary 2 *Fix $r > 0$. There is a unique threshold $\theta_1^r \in \Re$ that satisfies $\Pi(\theta_1^r) = 0$. The local bank lends if and only if the repayment probability θ is at least θ_1^r.*

3 Separating Equilibria with Remote Lending

We now show that for any $q \in [0,1)$, there are equilibria in which the local bank is limited to securitizing a proportion q of any loan it makes. Moreover, by increasing the remote bank's advantage at the securitization stage, this partial securitization property allows remote lending to occur.

Theorem 3 *Fix any $q \in [0,1)$.*

1. *In the subgame following the remote bank's offer of any interest rate r, there is an equilibrium in which the local bank never securitizes a proportion greater than q of any loan it makes.*

2. *Assume $a > \frac{\rho^{-1}-\delta}{1-\delta}$ and No Cream Skimming. There is a cutoff $\mu \in (0,1)$, which is nondecreasing in q, such that if the expected type $\overline{\theta}$ is at least μ, then the full game has*

[2] We assume that the local bank lends if it is indifferent.

an equilibrium in which only the remote bank competes and offers an interest rate r^* that deters the local bank from lending except to the highest type $\theta = 1$, of whose loan the local bank securitizes a proportion q.

Intuitively, the investors' pessimistic beliefs limit the local bank to a securitization proportion $q < 1$ whenever it does lend. These beliefs raise the deterring rate r^*, at which the local bank just willing to lend to the highest type. Able to collect the higher interest rate r^* without losing its best applicants to cream-skimming, the remote bank will lend if the expected type $\overline{\theta}$ is high enough.

4 Equilibria that Survive D1

We now consider equilibria that survive the D1 refinement of Banks and Sobel [2]. By protecting the local bank from pessimistic beliefs, D1 strengthens the local bank's competitive position to such an extent that the remote bank cannot compete.

Theorem 4 *Assume the refinement D1.*

1. *In the subgame following the remote bank's offer of any interest rate r:*

   (a) *If the local bank sometimes lends (if $\theta_1^r \leq 1$) then the signalling game has a unique equilibrium that survives D1. Investors respond to the quantity Q with the price $P^r(Q) = \frac{r\theta_1^r}{Q^{1-\delta}}$ and for all $\theta \in [\theta_1^r, 1]$, the local bank sells the quantity $Q^r(\theta) = \left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}$ and earns total profits $\Pi(\theta) = \delta r\theta - 1 + r(1-\delta)\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}$. If $\theta = \theta_1^r$, the local bank's profits are $\Pi(\theta_1^r) = r\theta_1^r - 1$, which equals zero and exceeds the remote bank's profits from offering r.*

   (b) *If the local bank never lends (if $\theta_1^r > 1$), r must be low enough that the the remote bank's profits from offering r are less than $\overline{\theta} - 1$ which is negative.*

2. *As the remote bank's profits in both cases are negative, it will not compete.*

The intuition runs roughly as follows. First suppose that the interest rate r is high enough that the local bank sometimes lends: $\theta_1^r \leq 1$ (part 1a). If the local bank then securitizes its entire loan, D1 forces investors to conclude that the applicant's success probability θ equals the local bank's lending threshold θ_1^r. Hence, the local bank can reap the entire gains from trade $(1-\delta)\, r\theta_1^r$ if it sells the whole loan. The local bank's profits from such a loan are thus $r\theta_1^r - 1$: the sum of its securitization profits $(1-\delta)\, r\theta_1^r$ and its discounted gross expected loan return $\delta r\theta_1^r$, less its unit cost of capital. As the local bank is indifferent between lending and not, its profits $r\theta_1^r - 1$ must be zero. But this leaves the remote bank with applicants $\theta < \theta_1^r$, for whom the joint payoff of the bank and investors, $r\theta - 1$, is negative. Since the investors must break even, the remote bank loses money. Accordingly, the remote bank will not make such an offer.

Now suppose the interest rate r is so low that the local bank never lends: $\theta_1^r > 1$ (part 1b). If the local bank unexpectedly issues a loan, by D1 investors believe that the loan type θ equals one as this type maximizes the profitability of the local bank's deviation. They thus assign the price r to the loan. As this exceeds the local bank's gross return $\delta r\theta$ from retaining the loan, it sells the entire loan, getting a profit of $r - 1$.[3] This cannot be positive since, by assumption, the remote bank never lends: $r \leq 1$. Hence, the remote bank's profits $r\overline{\theta} - 1$ are at most $\overline{\theta} - 1$, which is negative: the remote bank will not make such an offer either. Since the remote bank does not have a profitable offer, it will not compete.

4.1 D1: Discussion

As Fudenberg and Tirole [6] note, the beliefs restrictions that D1 imposes are strong and somewhat arbitrary. In our model, D1 seems to force investors to hold very specific and arbitrary beliefs about what a deviating local bank must think. In particular, for each loan type θ and securitization quantity Q, let $\underline{P}_Q^\theta$ be the minimum price that investors could assign to the loan that would make the local bank willing to deviate in the given way. D1

[3] This intuition assumes the securitization market functions: $a = 1$. The proof does not assume this.

states that investors believe that if the local bank lends and sells Q, investors conclude with certainty that the type θ is the type θ^* that yields the lowest such minimum price $\underline{P}_Q^{\theta}$. But for investors to believe this, they must think that the local bank expects P to equal the price $\underline{P}_Q^{\theta^*}$ at which it is willing to deviate only if its loan type is θ^*. For if the local bank expects the price P possibly to exceed $\underline{P}_Q^{\theta^*}$, it would want to deviate when its loan type exceeds θ^* as well. It is not clear why investors should think the local bank has these very specific beliefs.

A Proofs

Proof of Theorem 3: Fix a constant $q \in [0,1)$. We seek a separating equilibrium in which $Q^r(\theta_1^r) = q$ for any interest rate $r > 0$ for which $\theta_1^r \in [0,1]$. Let R be the set of such interest rates. We may assume w.l.o.g. that for any $r \in R$, if the local bank deviates to a quantity above q then investors conclude that the loan has no possibility of repayment.[4] This belief indeed prevents such a deviation since the resulting price P is zero.

Fix any $r \in R$. The price P that results from the quantity q is $\theta_1^r r$ and the local bank's payoff $U^r(\theta_1^r, q, \theta_1^r r)$ at $\theta = \theta_1^r$ is zero by Corollary 2, so $\theta_1^r = [aq + \delta(1-aq)]^{-1} r^{-1}$ by (1). Now consider the price function $P^r(Q) = \frac{r\theta_1^r q^{1-\delta}}{Q^{1-\delta}}$ and the quantity function $Q^r(\theta) = q\left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}$. The first order condition for an optimal quantity Q, evaluated using the given pricing function, is

$$\begin{aligned} 0 &= \frac{d}{dQ}U^r(\theta, Q, P^r(Q)) = \frac{d}{dQ}\left[\delta\theta r - 1 + aQ\left[P^r(Q) - \delta\theta r\right]\right] \\ &= a\left[P^r(Q) - \delta\theta r + Q\frac{dP^r(Q)}{dQ}\right] = a\delta r\left[\frac{\theta_1^r q^{1-\delta}}{Q^{1-\delta}} - \theta\right] \end{aligned}$$

which, at $Q = Q^r(\theta)$, is zero as $\frac{\theta_1^r q^{1-\delta}}{Q^r(\theta)^{1-\delta}} = \theta$. Moreover, for any θ, $U^r(\theta, Q, P^r(Q))$ is globally concave in Q as

$$\frac{d^2}{dQ^2}U^r(\theta, Q, P^r(Q)) = \frac{d^2}{dQ^2}\left[QP^r(Q)\right] = \frac{d^2}{dQ^2}\left[r\theta_1^r q^{1-\delta} Q^{\delta}\right]$$

[4] By Proposition 1, the local bank never securitizes more than q in equilibrium.

which is negative as $\delta \in (0,1)$. Finally, the pricing function reflects Bayesian updating since

$$P^r\left(Q^r\left(\theta\right)\right) = \frac{r\theta_1^r q^{1-\delta}}{Q^r\left(\theta\right)^{1-\delta}} = \frac{r\theta_1^r q^{1-\delta}}{\left(q\left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}\right)^{1-\delta}} = \theta r.$$

The local bank's equilibrium payoff to lending to any type $\theta \in [\theta_1^r, 1]$ is

$$\begin{aligned} U^r\left(\theta\right) &= U^r\left(\theta, Q^r\left(\theta\right), P^r\left(Q^r\left(\theta\right)\right)\right) = \delta\theta r - 1 + aQ^r\left(\theta\right)\left[P^r\left(Q^r\left(\theta\right)\right) - \delta\theta r\right] \\ &= \delta\theta r - 1 + aq\left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}\theta r\left[1-\delta\right] = \delta\theta r - 1 + aqr\left(1-\delta\right)\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}. \end{aligned}$$

We have produced a separating equilibrium in which the local bank sells up to the fixed proportion q, independent of r. We now turn to the remote bank's behavior: which r will it choose, and are its lending profits positive at this r? If so, there can be remote lending. If the remote bank offers r, it lends if and only if $\theta < \theta_1^r$ where

$$\theta_1^r = br^{-1} \tag{4}$$

and $b = \left[aq + \delta\left(1 - aq\right)\right]^{-1}$. Moreover, it securitizes the whole loan, getting the loan's expected value $rE\left[\theta | \theta < \theta_1^r\right]$. Let

$$r^* = b \tag{5}$$

be the deterring rate: if $r = r^*$, the local bank's threshold $\theta_1^r = 1$ so it is just deterred from competing. As the securitization market exists with probability a, the remote bank's expected payoff from offering r is $\left[a + \delta\left(1-a\right)\right] rE\left[\theta|\theta < \theta_1^r\right] - 1$.

Lemma 5 *Under No Cream Skimming, if the remote bank competes it will offer the rate* $\min\{\rho, r^*\}$ *and the local bank will not lend.*

Proof of Lemma 5: As the securitization market exists with probability a, the remote bank's expected payoff from offering r is

$$\left[a + \delta\left(1-a\right)\right] rE\left[\theta|\theta < \theta_1^r\right] - 1 = c^{-1}\int_{\theta=0}^{br^{-1}} \left(r\theta - c\right) dG_{\overline{\theta}}\left(\theta\right) \stackrel{d}{=} c^{-1} I\left(r\right)$$

where $c = \left[a + \delta\left(1-a\right)\right]^{-1}$. Suppose bank R does compete. Then it chooses an interest rate $r \leq \rho$ to maximize $I\left(r\right)$ and, moreover, $I\left(r\right) > 0$ at this optimal r. If $r < \min\{\rho, r^*\}$, then by (4) θ_1^r exceeds one so $I'(r) = \int_{\theta=0}^{1} r\theta dG_{\overline{\theta}}(\theta) > 0$: bank R's optimal r is at least $\min\{\rho, r^*\}$. It remains to show that its optimal r is at most $\min\{\rho, r^*\}$. If $r^* \geq \rho$, we are done since $\min\{\rho, r^*\} = \rho$ and, by assumption, bank R competes (chooses $r \leq \rho$). If $r^* < \rho$, consider any $r > r^*$. We will show, using No Cream Skimming, that if $I(r) > 0$ then $I'(r) < 0$: no such r can be optimal for bank R. By (4) and (5), $\theta_1^r = br^{-1} = r^*/r < 1$, so

$$I(r) = \int_{\theta=0}^{r^*/r} (r\theta - c)\, dG_{\overline{\theta}}(\theta).$$

With the change of variables $s = r\theta$, $I(r) = \frac{1}{r}\int_{s=0}^{r^*} (s-c)\, G'_{\overline{\theta}}\left(\frac{s}{r}\right) ds$. Thus,

$$I'(r) = -\frac{1}{r^2}\left(\int_{s=0}^{r^*} (s-c)\left[\frac{G'_{\overline{\theta}}\left(\frac{s}{r}\right) + G''_{\overline{\theta}}\left(\frac{s}{r}\right)\frac{s}{r}}{G'_{\overline{\theta}}\left(\frac{s}{r}\right)}\right] G'_{\overline{\theta}}\left(\frac{s}{r}\right) ds\right).$$

Changing variables back,

$$I'(r) = -\frac{1}{r}\left(\int_{\theta=0}^{r^*/r} (r\nu - c)\left[\frac{G'_{\overline{\theta}}(\nu) + G''_{\overline{\theta}}(\nu)\,\nu}{G'_{\overline{\theta}}(\nu)}\right] dG_{\overline{\theta}}(\nu)\right).$$

For any functions $\varphi_0(\theta)$ and $\varphi_1(\theta)$, let $E^*(\varphi_0)$ and $Cov^*(\varphi_0, \varphi_1)$ denote the expectation of φ_0 and covariance of φ_0 and φ_1, both conditioned on θ lying in $[0, r^*/r]$. Then $I'(r) = -\frac{1}{r}E^*(xy)\, G_{\overline{\theta}}(r^*/r)$, where $x(\theta) = r\theta - c$ and $y(\theta) = \frac{G'_{\overline{\theta}}(\theta) + G''_{\overline{\theta}}(\theta)\theta}{G'_{\overline{\theta}}(\theta)}$. By definition of covariance, $E^*(xy) = Cov^*(x,y) + E^*(x)\, E^*(y)$. By assumption $E^*(x) = \frac{I(r)}{G_{\overline{\theta}}(r^*/r)} > 0$. By No Cream Skimming, $y(\nu)$ is positive and nondecreasing in ν, so $E^*(y) > 0$ and $Cov^*(x,y) \geq 0$. Thus $E^*(xy) > 0$, so $I'(r) < 0$ as claimed. Q.E.D.$_{\text{Lemma 5}}$

Thus, if the local bank competes, its profits are $\Pi_R(r) = [a + \delta(1-a)] \min\{\rho, r^*\}\, \overline{\theta} - 1$ by Lemma 5. It will compete if this is positive: if the unconditional expected success probability $\overline{\theta}$ exceeds the threshold

$$\begin{aligned}
[\min\{\rho, b\}]^{-1} &= [a + \delta(1-a)]^{-1}\left[\min\left\{\rho, [aq + \delta(1-aq)]^{-1}\right\}\right]^{-1} \\
&= \max\left\{\frac{1}{\rho[a + \delta(1-a)]}, \frac{aq + \delta(1-aq)}{a + \delta(1-a)}\right\}.
\end{aligned}$$

The numerator in the second ratio is increasing in q as $\delta < 1$; the denominator in the second ratio equals the numerator evaluated at $q = 1$. Hence, the second ratio is less than one for any $q < 1$. The first ratio is less than one as long as $a > \frac{\rho^{-1}-\delta}{1-\delta}$. Thus, as long as this condition on a holds, there exist distributions of success probabilities θ that prompt the remote bank to compete. Q.E.D.$_{\text{Theorem 3}}$

Proof of Claim 1: Assume $\theta' > \theta'' \geq \theta_1^r$; let $Q' = Q^r(\theta')$, $Q'' = Q^r(\theta'')$, $P' = P^r(Q')$, and $P'' = P^r(Q'')$. Then incentive compatibility requires that $\Pi(\theta') = U^r(\theta', Q', P') \geq U^r(\theta', Q'', P'')$ and $\Pi(\theta'') = U^r(\theta'', Q'', P'') \geq U^r(\theta'', Q', P')$. Hence,

$$U^r(\theta', Q', P') - U^r(\theta'', Q', P') \geq U^r(\theta', Q'', P'') - U^r(\theta'', Q'', P'')$$

By (1), $(\theta' - \theta'')(Q'' - Q') \geq 0$ so $Q'' \geq Q'$ as claimed. Moreover, by (1), $\Pi(\theta') - \Pi(\theta'')$ is bounded above by $U^r(\theta', Q'', P'') - U^r(\theta'', Q'', P'') = \delta(\theta' - \theta'') r [1 - aQ'']$ and below by $U^r(\theta', Q', P') - U^r(\theta'', Q', P') = \delta(\theta' - \theta'') r [1 - aQ']$. Both bounds are positive and at most $\delta\rho|\theta' - \theta''|$, so $\Pi()$ is continuous and increasing in θ. Finally as $Q' \in [0, 1]$, $\frac{\Pi(\theta')-\Pi(\theta'')}{\theta'-\theta''} \geq \delta r(1-a) > 0$ which implies the two limiting properties. Q.E.D.$_{\text{Claim 1}}$

Proof of Theorem 4: Part 1. Fix an equilibrium. Let R be the set of interest rates r for which the local bank ever lends: for which $\theta_1^r \leq 1$. For any $r \in R$, let q^r be the maximum the local bank ever securitizes conditional on lending: $q^r = \max_{\theta \geq \theta_1^r} Q(\theta)$. We first show that D1 rules out equilibria in which the local bank never securitizes conditional on lending.

Lemma 6 *In any equilibrium that survives D1, for any $r \in R$, $q^r > 0$.*

Proof of Lemma 6: W.l.o.g. we can assume any positive quantity Q leads to the belief that $\theta = 0$, so $P = 0$. In this equilibrium, $\delta\theta_1^r r = 1$, so $\theta_1^r = 1/\delta r$. Fix a quantity $Q > 0$. For each price P, and each type θ, let $\overline{\mu}(\theta, P)$ be the local bank's set of optimal probabilities of lending and then selling Q, vs. sticking to the equilibrium, if its type is θ and the price it anticipates from this deviation is P. There are two cases.

1. If $\theta \geq \theta_1^r = 1/\delta r$, the local bank lends in equilibrium, getting $\delta\theta r - 1$. So its relative payoff from this deviation is

$$U^r(\theta, Q, P) - (\delta\theta r - 1) = aQ[P - \delta\theta r].$$

This is nonnegative iff $\theta \leq \frac{P}{\delta r} = \gamma_P$, which is possible only if $P \geq 1$. Intuitively, we know that $\delta\theta r \geq 1$, as the local bank is willing to lend without securitizing. But for it to be willing to securitize any of the loan, it must be that the price P that it gets is not less than the discounted expected return $\delta\theta r$ of the loan. Hence, $P \geq 1$. For $P > 1$,

$$\overline{\mu}(\theta, P) = \begin{cases} 1 & \text{if } \frac{1}{\delta r} \leq \theta < \gamma_P \\ [0,1] & \text{if } \theta = \gamma_P \\ 0 & \text{if } \theta > \gamma_P \end{cases};$$

If $P = 1$, $\overline{\mu}(\theta, P) = \begin{cases} [0,1] & \text{if } \theta = \gamma_P = \frac{1}{\delta r} \\ 0 & \text{if } \theta > \gamma_P = \frac{1}{\delta r} \end{cases}$; if instead $P < 1$, then $\overline{\mu}(\theta, P) = 0$ for all $\theta \geq \theta_1^r$.

2. If $\theta \leq \theta_1^r = 1/\delta r$, the local bank is willing not to lend in equilibrium. So its relative payoff from this deviation is

$$U^r(\theta, Q, P) = -1 + \delta\theta r[1 - aQ] + aQP.$$

This is nonnegative iff $\theta \geq \frac{1-aQP}{\delta r[1-aQ]} = \lambda_P^Q$, which is possible only if $\lambda_P^Q \leq \frac{1}{\delta r} = \theta_1^r$ or, equivalently, if $1 - aQP \leq 1 - aQ$ or, equivalently, if $P \geq 1$ (the same condition as in case 1). In this case, if $P > 1$,

$$\overline{\mu}(\theta, P) = \begin{cases} 1 & \text{if } \lambda_P^Q < \theta \leq \frac{1}{\delta r} \\ [0,1] & \text{if } \theta = \lambda_P^Q \\ 0 & \text{if } \theta < \lambda_P^Q \end{cases}$$

while if $P = 1$, $\overline{\mu}(\theta, P) = \begin{cases} [0,1] & \text{if } \theta = \lambda_P^Q = \frac{1}{\delta r} \\ 0 & \text{if } \theta < \lambda_P^Q \end{cases}$. Finally, if $P < 1$, then $\overline{\mu}(\theta, P) = 0$ for all $\theta < \theta_1^r$.

Collecting cases 1 and 2, if $P > 1$, then $\lambda_P^Q < \frac{1}{\delta r} < \gamma_P$ so

$$\overline{\mu}(\theta, P) = \begin{cases} 1 & \text{if } \theta \in \left(\frac{1-aQP}{\delta r[1-aQ]}, \frac{P}{\delta r}\right) \stackrel{d}{=} \left(\lambda_P^Q, \gamma_P\right) \\ [0,1] & \text{if } \theta \in \left\{\lambda_P^Q, \gamma_P\right\} \\ 0 & \text{if } \text{otherwise} \end{cases}$$

while if $P=1$, then $\lambda_P^Q = \frac{1}{\delta r} = \gamma_P$ so $\overline{\mu}(\theta,P) = \begin{cases} [0,1] & \text{if} \quad \theta = \frac{1}{\delta r} \\ 0 & \text{if} \quad \theta \neq \lambda_P^Q \end{cases}$, and if $P<1$, then $\overline{\mu}(\theta,P)=0$ for all θ: no bank wants to deviate.

Finally, D1 states that if each price P that makes type θ (say) weakly prefer to deviate to Q also makes type θ' strictly prefer to deviate to Q, then on seeing Q the investors believe that the local bank's type is θ'. Hence, they believe that $\theta = \min\{1, 1/\delta r\}$, regardless of Q, and thus assign the price $P = r\theta = \min\{r, 1/\delta\}$. As $r \in R$, consider $\theta = \theta_1^r = 1/\delta r \leq 1$. If the local bank sells a proportion Q of the loan, it gains $Q[P - \delta r\theta] = Q[1/\delta - 1] > 0$. Hence, $q^r > 0$ as claimed. Q.E.D.$_{\text{Lemma 6}}$

We now turn to equilibria of the securitization subgame in which $q^r \in (0,1]$. We show that D1 implies that $q^r = 1$ and that these equilibria have the DeMarzo-Duffie form.

Lemma 7 *Consider an equilibrium that survives D1. Suppose that for some given* $r \in R$*,* $q^r \in (0,1]$*. Then* $q^r = 1$*,* $P^r(Q) = \frac{r\theta_1^r}{Q^{1-\delta}}$*, and for all* $\theta \in [\theta_1^r, 1]$*,* $Q^r(\theta) = \left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}$ *and* $\Pi(\theta) = \delta r\theta - 1 + r(1-\delta)\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}$.

Proof of Lemma 7: By Claim 1, $Q^r(\theta_1^r) = q^r$. By Corollary 2, $\Pi(\theta_1^r) = 0$. The local bank's signal has two components: the decision to lend, and the quantity Q to securitize. Let 1_{lend} equal one if the local bank lends and zero otherwise. The local bank's realized profit from its choice given the repayment probability θ and the anticipated security price P is

$$\widehat{U}^r(\theta, (1_{\text{lend}}, Q), P) = U^r(\theta, Q, P) * 1_{\text{lend}}$$

where U is defined in (1). Hence, the analogue to the Spence-Mirrlees sorting condition (Fudenberg and Tirole [6, ch. 11]) in our model is

$$-\frac{\partial U^r/\partial Q}{\partial U^r/\partial P} = \begin{cases} -\frac{a[P-\delta\theta r]}{aQ} = \frac{\delta\theta r - P}{Q} \text{ which is increasing in } \theta & \text{if} \quad 1_{\text{lend}} = 1 \\ \text{undefined} & \text{if} \quad 1_{\text{lend}} = 0 \end{cases}$$

Lemma 11.2 of Fudenberg and Tirole [6] can now be adapted to our model as follows.

Lemma 8 *Let $\theta'' > \theta' \geq \theta_1^r$. Also assume that when $\theta = \theta''$, the local bank lends and sells Q' with positive probability in equilibrium. Then D1 implies that $\mu(\theta'|Q'') = 0$ for all $Q'' < Q'$.*

Proof of Lemma 8: Fix an equilibrium $(\sigma_1^*(1_{\text{lend}}, Q|\theta), \sigma_2^*(P|1_{\text{lend}}, Q))$ such that (a) type θ'' lends and sells Q' with positive probability ($\sigma_1^*(1, Q'|\theta'') > 0$) and (b) type θ' sometimes lends (there is a Q for which $\sigma_1^*(1, Q|\theta') > 0$). Let $P^*((1_{\text{lend}}, Q))$ denote the investors' equilibrium price in response to the local bank's action $(1_{\text{lend}}, Q)$. For each $Q'' < Q'$ and every θ for which the local bank sometimes lends, let $\widehat{P}(\theta)$ in the set $\Pi = [0, r]$ of optimal prices for any type θ satisfy $U^r\left(\theta, (1, Q''), \widehat{P}(\theta)\right) = U^{r*}(\theta)$ (the equilibrium payoff of the local bank when the type is θ). If no such action $\widehat{P}(\theta)$ exists, let $\widehat{P}(\theta) = \infty$. We claim that $\widehat{P}(\theta') > \widehat{P}(\theta'')$. For assume not: $\widehat{P}(\theta') \leq \widehat{P}(\theta'')$. By definition of $\widehat{P}(\theta)$, type θ'' is indifferent between the action pair $A = ((1, Q'), P^*((1, Q')))$ and $B = \left((1, Q''), \widehat{P}(\theta'')\right)$. By Spence-Mirrlees, conditional on $1_{\text{lend}} = 1$, the indifference curve in (Q, P) space of type θ' is steeper than that of type θ'' at any point. But since $\widehat{P}(\theta') \leq \widehat{P}(\theta'')$, the indifference curve of type θ'' lies above the indifference curve of type θ' at $Q = Q''$. Thus, these two indifference curves cannot intersect at any $Q < Q''$. Therefore, type θ' strictly prefers $((1, Q'), P^*((1, Q')))$ to his equilibrium strategy - and thus will deviate from the assumed equilibrium. This is a contradiction. (I.e., it's not an equilibrium after all.) We have shown that $\widehat{P}(\theta') > \widehat{P}(\theta'')$. Thus, the set $\left[\widehat{P}(\theta'), r\right]$ of prices P that make type θ' willing to sell the quantity Q'' is strictly contained in the set $\left(\widehat{P}(\theta''), r\right]$ of prices that make type θ'' strictly prefer to sell the quantity Q''. Hence, by D1, on seeing the local bank lend and then sell Q'', investors must assign probability zero to the type being θ'. Q.E.D.$_{\text{Lemma 8}}$

Lemma 9 *Any equilibrium in which two or more types θ lend and assign positive probability to the same quantity $Q^* > 0$ fails criterion D1.*

Proof of Lemma 9: Suppose not. Let θ^* be the highest type that ever sells Q^*, and let $\overline{\theta}$ be the investors' posterior over θ when they see the quantity Q^*; hence, the price that results from Q^* is $r\overline{\theta}$. Since there is some pooling at Q^*, $\overline{\theta} < \theta^*$. Suppose type θ^* deviates to the quantity $Q^* - \iota$ for any $\iota \in (0, Q^*)$. By the lemma, the investors' posterior over θ when they see the quantity $Q^* - \iota$ is at least θ^*. Hence, by switching from Q^* to $Q^* - \iota$ the local bank can raise the price by at least $\alpha = r\left[\theta^* - \overline{\theta}\right] > 0$, independent of ι. Its change in its profits is at least

$$
\begin{aligned}
& U^r\left(\theta^*, Q^* - \iota, r\overline{\theta} + \alpha\right) - U^r\left(\theta^*, Q^*, r\overline{\theta}\right) \\
= \; & \delta\theta^* r\left[1 - a\left(Q^* - \iota\right)\right] + a\left(Q^* - \iota\right)\left(r\overline{\theta} + \alpha\right) - \left(\delta\theta^* r\left[1 - aQ^*\right] + aQ^* r\overline{\theta}\right) \\
= \; & \delta\theta^* r a\iota + a\left[-\iota r\overline{\theta} + Q^*\alpha - \iota\alpha\right]
\end{aligned}
$$

which must be positive if $\iota < \frac{Q^*\alpha}{\alpha + r\overline{\theta}} \in \Re_{++}$. Q.E.D.$_{\text{Lemma 9}}$

Lemma 10 *The function Q^r is continuous.*

Proof of Lemma 10: Suppose it is not. By Claim 1, its range must have a gap $G \subset (0,1)$ of the form $(a, b]$ or $[a, b)$. Let G be maximal in the sense that for all $\iota > 0$, some quantities in $(a - \iota, a]$ and in $[b, b + \iota)$ are offered in equilibrium. If the gap is $(a, b]$, let θ' be the type that sells the quantity b in equilibrium; if $G = [a, b)$, let θ' sell $Q = a$. Since $G \subset (0,1)$, $\theta' > 0$, so type θ' is unique by Lemma 9. Since the support of the types θ is connected and by Claim 1, for any $\varepsilon > 0$ there is an $\iota > 0$ such that (a) any type θ that sells any quantity Q_0 in $(a - \iota, a]$ must lie in the interval $[\theta', \theta' + \varepsilon/2)$ and (b) any type θ' that sells any quantity Q in $[b, b + \iota)$ must lie in the interval $(\theta' - \varepsilon/2, \theta']$. Now pick a type θ that sells a quantity Q_0 in $(a - \iota, a]$. Suppose θ deviates to the quantity $Q_1 = b - \varepsilon$. Let the price that follows the quantity Q_i be P_i for $i = 0, 1$. Since there is no pooling except at $Q = 0$, $P_0 = \theta r$. The payoff from this deviation is thus

$$
\begin{aligned}
U^r\left(\theta, Q_1, P_1\right) - U^r\left(\theta, Q_0, P_0\right) &= a\left[Q_1\left(P_1 - P_0\right) + \theta r\left(Q_1 - Q_0\right)(1 - \delta)\right] \\
&\geq a\left[-\varepsilon r + \theta r\left(b - a - \varepsilon\right)(1 - \delta)\right]
\end{aligned}
$$

as $Q_1\left(P_1 - P_0\right) \geq -\varepsilon r$ (since $Q_1 \leq 1$ and the investors' posterior over θ falls by at most ε), while $Q_1 - Q_0 \geq b - a - \varepsilon$. Hence, the deviation is profitable as long as $\varepsilon \in \left(0, \frac{\theta r(b-a)(1-\delta)}{r + \theta r[1-\delta]}\right)$.
Q.E.D.$_{\text{Lemma 10}}$

Lemma 11 $Q^r(1) > 0$.

Proof of Lemma 11: Suppose not. Let $\overline{\theta}$ be the lowest type θ for which $Q^r(\theta) = 0$. This type exists by Lemma 10 and exceeds θ_1^r as $Q^r(\theta_1^r) = q^r > 0$. Let the price that results from the quantity 0 be P_0. By Lemma 10, for any $\varepsilon > 0$ there is an $\iota > 0$ such that if the local bank (of type $\overline{\theta}$) deviates to some $Q_1 \in (0, \iota)$, the investors believe that $\theta \in (\overline{\theta} - \varepsilon, \overline{\theta})$. Let the price that results from this deviation be $P_1 \geq r(\overline{\theta} - \varepsilon)$. The local bank's payoff changes by

$$\begin{aligned} U^r(\overline{\theta}, Q_1, P_1) - U^r(\overline{\theta}, 0, P_0) &= \left[\delta\overline{\theta}r[1 - aQ_1] + aQ_1P_1\right] - \delta\overline{\theta}r \\ &= aQ_1(P_1 - \delta\overline{\theta}r) \geq aQ_1r((1-\delta)\overline{\theta} - \varepsilon) \end{aligned}$$

which is positive if $\varepsilon \in (0, (1-\delta)\overline{\theta})$. Hence, the local bank will deviate, a contradiction. Q.E.D.$_{\text{Lemma 11}}$

By Lemma 9, only the type θ_1^r sells the quantity $q^r > 0$. Hence $P^r(q^r) = r\theta_1^r$, so

$$0 = U^r(\theta_1^r, q^r, P) = \delta\theta_1^r r - 1 + aq^r[P - \delta\theta_1^r r] = \theta_1^r(\delta r + aq^r r[1-\delta]) - 1$$

and thus

$$\theta_1^r = r^{-1}(\delta + aq^r[1-\delta])^{-1}. \tag{6}$$

By Lemmas 9, 10, and 11, the function $Q^r : [\theta_1^r, 1] \to [0, 1]$ is continuous and strictly decreasing, and its range is a subinterval of $(0, 1)$. Hence, it has a continuous, strictly decreasing inverse Ψ. For all $\theta \in [\theta_1^r, 1]$, on seeing $Q = Q^r(\theta)$ investors know the type is $\Psi(Q)$ and thus assign the price $P^r(Q) = r\Psi(Q)$. Let $q^r > Q_1 > Q_0 > Q^r(1)$. Hence, there are $\theta_0, \theta_1 \in (\theta_1^r, 1)$, $\theta_0 > \theta_1$, such that, for $i = 0, 1$, $Q_i = Q^r(\theta_i)$. Let $P_i = r\theta_i = P^r(Q_i)$. By incentive compatibility,

$$0 \geq U^r(\theta_0, Q_1, P_1) - U^r(\theta_0, Q_0, P_0) = a(Q_1(P_1 - P_0) + (1-\delta)P_0[Q_1 - Q_0])$$

and

$$0 \geq U^r(\theta_1, Q_0, P_0) - U^r(\theta_1, Q_1, P_1) = a(Q_0(P_0 - P_1) + (1-\delta)P_1[Q_0 - Q_1]).$$

Rearranging, $P_1 - P_0 \leq -(1-\delta)\frac{P_0}{Q_1}[Q_1 - Q_0]$ and $P_1 - P_0 \geq -(1-\delta)\frac{P_1}{Q_0}[Q_1 - Q_0]$. Hence,

$$(1-\delta)\left[\frac{P_0}{Q_0} - \frac{P_1}{Q_0}\right] \leq \frac{P_1 - P_0}{Q_1 - Q_0} - \left[-(1-\delta)\frac{P_0}{Q_0}\right] \leq (1-\delta)\left[\frac{P_0}{Q_0} - \frac{P_0}{Q_1}\right].$$

Since $P^r()$ is continuous and $Q^r(1) > 0$, this implies that as $|Q_1 - Q_0|$ shrinks to zero, so does $\frac{P_1 - P_0}{Q_1 - Q_0} - \left[-(1-\delta)\frac{P_0}{Q_0}\right]$. Hence, $P^r()$ is differentiable at all points $Q \in (Q^r(1), q^r)$ and satisfies

$$\frac{dP^r(Q)}{dQ} = -\frac{P^r(Q) - \delta\theta r}{Q} = -(1-\delta)\frac{P^r(Q)}{Q}. \tag{7}$$

Since the function $\frac{1-\delta}{Q}$ is continuous in $Q \in [Q^r(1), q^r]$, (7) has a unique solution on this interval (see, e.g., Apostol [1, ch. 8.3]). It must be $P^r(Q) = \frac{r\theta_1^r (q^r)^{1-\delta}}{Q^{1-\delta}}$, as this function satisfies (7) as well as the boundary condition $P^r(q^r) = r\theta_1^r$. Inverting this function, $Q^r(\theta) = q^r\left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}$. Hence,

$$P^r(Q^r(\theta)) = \frac{r\theta_1^r (q^r)^{1-\delta}}{Q^r(\theta)^{1-\delta}} = \frac{r\theta_1^r (q^r)^{1-\delta}}{\left(q^r\left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}\right)^{1-\delta}} = \theta r$$

as required for separation. This permits us to compute the equilibrium payoffs of all types $\theta \in [\theta_1^r, 1]$:

$$\begin{aligned} U^r(\theta) &= U^r(\theta, Q^r(\theta), P^r(Q^r(\theta))) = \delta\theta r - 1 + aQ^r(\theta)[P^r(Q^r(\theta)) - \delta\theta r] \\ &= \delta\theta r - 1 + aq^r\left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}\theta r[1-\delta] \\ &= \delta\theta r - 1 + aq^r r(1-\delta)\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}. \end{aligned}$$

We can now check D1. We first compute the payoff change of each type $\theta \in [0,1]$ from (lending and) deviating to each quantity $Q \in (q^r, 1]$ when each price P is anticipated. For $\theta \leq \theta_1^r$, this payoff change is

$$U^r(\theta, Q, P) = \delta\theta r - 1 + aQ[P - \delta\theta r]$$

which is positive if and only if $P > \frac{1-(1-aQ)\delta\theta r}{aQ}$. As the right hand side of this inequality is decreasing in θ, the inequality holds for the widest range of prices P when θ takes its maximum value of θ_1^r. Hence, investors cannot believe that $\theta < \theta_1^r$. For θ in $[\theta_1^r, 1]$ the payoff change from the deviation is

$$\begin{aligned} U^r(\theta, Q, P) - U^r(\theta) &= \delta\theta r - 1 + aQ[P - \delta\theta r] - \left[\delta\theta r - 1 + aq^r r(1-\delta)\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}\right] \\ &= a\left[Q[P - \delta\theta r] - q^r r(1-\delta)\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}\right] \end{aligned}$$

which is positive if and only if $P > r\left[\delta\theta + (1-\delta)\frac{q^r}{Q}\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}\right]$. The right hand side of this inequality is of the form $a\theta + b\theta^{-\frac{\delta}{1-\delta}}$ where $a = r\delta$ and $b = r(1-\delta)\frac{q^r}{Q}\theta_1^{r\frac{1}{1-\delta}}$. The first derivative is $a - \frac{\delta}{1-\delta}b\theta^{-\frac{1}{1-\delta}}$ and second derivative is $\frac{\delta}{(1-\delta)^2}b\theta^{-\frac{2-\delta}{1-\delta}} > 0$. Hence, the function is strictly convex and has a unique minimum, which is

$$\theta(Q) = \left(\frac{\delta}{1-\delta}\frac{b}{a}\right)^{1-\delta} = \left(\frac{\delta}{1-\delta}\frac{(1-\delta)\frac{q^r}{Q}(\theta_1^r)^{\frac{1}{1-\delta}}}{\delta}\right)^{1-\delta} = \left(\frac{q^r}{Q}\right)^{1-\delta}\theta_1^r,$$

which in turn is strictly less than θ_1^r. Hence, the function is increasing in $\theta \geq \theta_1^r$. Thus, D1 implies that investors cannot believe that $\theta > \theta_1^r$. We have shown that following a deviation to $Q > q^r$, investors must believe that $\theta = \theta_1^r$. This means that type $\theta = \theta_1^r$, which gets zero in equilibrium, gets

$$\begin{aligned} U^r(\theta_1^r, 1, r\theta_1^r) &= \delta\theta_1^r r - 1 + a[\theta_1^r r - \delta\theta_1^r r] = \theta_1^r r[\delta + a(1-\delta)] - 1 \\ &= \frac{\delta + a(1-\delta)}{\delta + aq^r(1-\delta)} - 1 \end{aligned}$$

(by (6)) which exceeds zero unless $q^r = 1$. Thus, the only equilibrium that survives D1 is

$$P^r(Q) = \frac{r\theta_1^r}{Q^{1-\delta}},\ Q^r(\theta) = \left(\frac{\theta_1^r}{\theta}\right)^{\frac{1}{1-\delta}}.$$

The local bank's total profits $\Pi(\theta)$ equal the discounted gross loan return $\delta r\theta$ less the unitary cost of capital plus its securitization profits:

$$\Pi(\theta) = \delta r\theta - 1 + [P^r(Q^r(\theta)) - \delta r\theta]Q^r(\theta) = \delta r\theta - 1 + r(1-\delta)\left(\frac{\theta_1^r}{\theta^\delta}\right)^{\frac{1}{1-\delta}}$$

as claimed. Q.E.D.$_{\text{Lemma 7}}$

Lemmas 6 and 7 together imply part 1 of the theorem. We now turn to part 2. First, the remote bank will not offer an interest rate $r \in R$ (for which $\theta_1^r \leq 1$). For suppose it does. By Lemma 7, if $\theta = \theta_1^r$, the local bank gets total profits $0 = \Pi(\theta) = r\theta_1^r - 1$ and hence $\theta_1^r = 1/r$. The remote bank lends only if $\theta < \theta_1^r$. In this case, since there is symmetric information between the remote bank and investors, the remote bank sells its loan and gets the price $rE[\theta|\theta < 1/r] < 1$. Since its cost of capital is one, it loses money on the loan. Hence it will not offer such an interest rate, as claimed.

Finally, the remote bank will not offer an interest rate $r \notin R$ (for which $\theta_1^r > 1$) either. For suppose it does. D1 states that if each price P that makes type θ (say) weakly prefer to deviate to $(1_{\text{lend}}, Q) = (1, Q)$ also makes type θ' strictly prefer to deviate to $(1, Q)$, then on seeing $(1, Q)$ the investors believe that the local bank's type is θ'. Now suppose that $\theta_1^r > 1$ and the local bank unexpectedly chooses action $(1, Q)$: it lends and offers a proportion Q for sale. Let it anticipate the price P. Its payoff from this deviation is $U^r(\theta, Q, P) = -1 + \delta\theta r[1 - aQ] + aQP$ which is strictly increasing in θ for any given Q and P as $a < 1$. Thus, by D1, investors must believe that $\theta = 1$, regardless of Q. They thus assign the price r. This is more than the local bank can get by retaining the loan, so its best such deviation sets $Q = 1$. Hence, its deviation profits are $U^r(\theta, 1, r) = r[\delta\theta(1-a) + a] - 1$. The cutoff θ_1^r sets this to zero: $\theta_1^r = \frac{r^{-1}-a}{\delta(1-a)}$ which, by assumption, exceeds one: $r < [a + (1-a)\delta]^{-1}$. Thus, the remote bank's profits from offering r are

$$r\overline{\theta}(a + (1-a)\delta) - 1 < \overline{\theta} - 1 < 0:$$

the remote bank will not offer such an interest rate r either. This proves part 2. Q.E.D.$_{\text{Theorem 4}}$

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