



Recurrent crises in global games

David M. Frankel*

Department of Economics, Iowa State University, Ames, IA 50011, United States

ARTICLE INFO

Article history:

Received 11 December 2011

Received in revised form

15 April 2012

Accepted 25 July 2012

Available online 6 August 2012

Keywords:

Global games

Recurrent crises

Shocks

Monopoly

Network externalities

ABSTRACT

Global games have unique equilibria in which aggregate behavior changes sharply when an underlying random fundamental crosses some threshold. This property relies on the existence of dominance regions: all players have a highest and lowest action that, for some fundamentals, is strictly dominant. But if the fundamental follows a random walk, it eventually spends nearly all of its time in these regions: crises gradually disappear. We obtain recurring crises by adding a single large player who lacks dominance regions. We also show that in order to obtain recurring crises, one must either relax dominance regions or restrict to fundamentals that continually return to or cross over a fixed region.

© 2012 Elsevier B.V. All rights reserved.

1. Introduction

Large shifts in economic behavior often occur with little or no apparent change in economic fundamentals. Researchers have increasingly relied on global games (Carlsson and van Damme, 1993) to explain such shifts. These games have been used to model international contagion and bank runs (Goldstein and Pauzner, 2004, 2005), currency crises and market crashes (Morris and Shin, 1998, 2004), investment cycles (Chamley, 1999), merger waves (Toxvaerd, 2008), and securitization booms (Jin, 2011).¹

In a global game, each player observes a slightly noisy private signal of an exogenous, random fundamental. Players then choose actions from a bounded set: whether or not to invest, attack a currency, withdraw funds from a bank, etc. For extreme values of the fundamental, players are assumed to have strictly dominant actions. A contagion argument that begins in these dominance regions then establishes that there is a unique equilibrium, in which a large shift in behavior occurs when the fundamental crosses a given threshold(s).²

An important question is, what happens if such games are repeated? In a dynamic setting, for fluctuations to recur, the

fundamental must remain near the threshold, or at least return to it from time to time. This will not happen if the fundamental follows a random walk: players will eventually remain in one dominance region or another for arbitrarily long stretches of time. Researchers have addressed this problem by restricting the fundamental to a finite interval (Chamley, 1999; Oyama, 2004) or by assuming that the fundamental is mean-reverting (Burdzy and Frankel, 2005; Frankel and Pauzner, 2000). But while this may be a good description of some fundamental variables (e.g., tastes), it is likely that others (e.g., technology) are subject to shocks that are, to some degree, permanent.

We propose a solution that may be useful in some contexts: introducing a large player who lacks dominance regions. We begin with the global investment game of Morris and Shin (2000a, pp. 59 ff.), in which a continuum of agents simultaneously choose whether or not to invest. An agent's payoff from investing is increasing in the proportion of agents who invest and in an unknown random fundamental. Each agent receives a slightly noisy private signal of the fundamental. There is a unique equilibrium, in which each agent invests if and only if her signal exceeds a common threshold.

We modify this model by assuming that an agent who invests must buy a unit of a critical input from a monopoly supplier. The supplier first announces a price. Agents then see their signals as before and decide whether or not to invest.

In this setting, the supplier faces a tradeoff. A higher price leads to higher profits if agents invest (the benefit), but also raises their threshold, making them less likely to do so (the cost).³ At the

* Tel.: +1 515 294 6263; fax: +1 515 294 0221.

E-mail address: dfrankel@iastate.edu.

¹ Global-games-type arguments have also been applied to continuous-time stochastic games to study industrial development and neighborhood tipping (Frankel and Pauzner, 2000, 2002), search-friction-driven business cycles (Burdzy and Frankel, 2005), and currency crises (Guimaraes, 2006).

² Multiple thresholds can occur if agents have more than two actions or are heterogeneous (Frankel et al., 2003). Limitations on the uniqueness result are studied in Angeletos et al. (2006, 2007) and Angeletos and Werning (2006).

³ Intuitively, signals that previously justified investment no longer do so since the input is more costly.

supplier's optimal price, this benefit and cost exactly cancel. The cost is proportional to the prior density of the fundamental at the agents' investment threshold. Hence, the supplier's optimal price must yield a threshold at which the fundamental has a nontrivial density. This generally yields a nonnegligible probability that the fundamental lies below the threshold as well. That is, the supplier optimally tolerates a nontrivial crisis risk in order to get a higher price when a crisis is averted.

We then consider the effects of past positive shocks to the fundamental Z . We assume that $\ln Z = \mu + \sigma u$, where μ and $\sigma > 0$ are parameters and u has a fixed distribution with zero mean and unit variance. In the limit as the mean μ of the log fundamental goes to infinity, the crisis risk converges to a strictly positive constant, which is increasing in the dispersion parameter σ . Hence, long-run growth in the fundamental does not lead crises to vanish. In this way, the addition of a single large player without dominance regions can ensure that crises recur.

We then modify the model by assuming that the supplier observes a public signal of the fundamental before choosing a price. An increase in the precision of this signal leads to a lower crisis risk and higher social welfare. Intuitively, with a less noisy signal, the supplier can more easily extract the agents' informational rents without causing a crisis. However, in contrast with the standard monopoly model, the agents also have *strategic rents*: by virtue of their coordination problem, their valuation of the input strictly exceeds their reservation price. As the supplier's information becomes arbitrarily precise, the agents retain these strategic rents even as they lose their informational rents.

This model can also be seen as a contribution to the literature on goods that display network externalities. When payoffs are common knowledge, the buyers' purchase decision typically displays multiple equilibria (e.g., Farrell and Katz, 1998). We instead assume that the buyers play a global game, so the equilibrium is unique.

This is also the approach of Argenziano (2008). She considers two firms that sell competing goods which display network externalities. A noisy public signal of a random fundamental is first realized. This signal represents the relative quality of one good versus the other. Each firm then announces a price. Buyers then see their valuations, which are the sum of the fundamental and an idiosyncratic taste term. They then choose which good to buy. Given the two prices, there is a unique equilibrium by the usual global games reasoning.

Argenziano shows that a firm responds to a positive public signal of the relative quality of its good by raising its price. As a result, its increase in market share is inefficiently small. Analogously, the supplier in our model responds to a positive shock by raising its price, leading to an inefficiently small decline in the crisis risk. Relative to Argenziano (2008), our finding that monopoly rent extraction can yield recurrent crises appears to be new, as well as our observation that the agents have strategic rents that are not dissipated as the supplier becomes well informed about the fundamental. In Argenziano (2008), in contrast, these rents coexist with rents that result from price competition between the firms.

Summarizing, crises can recur if either some players lack dominance regions or the fundamental jumps back and forth over the crisis threshold. In Section 3, we show that these are the only two solutions, in the following sense. We consider a continuum of anonymous players who interact repeatedly over time. A player's payoff depends on a vector of exogenous, random private signals (e.g., of common fundamentals). Each player has at least one signal that displays dominance regions, is asymptotically unpredictable, and does not display arbitrarily large reversals (back and forth jumps). In this setting, the crisis frequency must asymptotically approach zero.

2. A simple model with recurring crises

All participants are risk-neutral and fully rational. There is a continuum of agents, each of whom can either do nothing, getting a payoff of zero, or invest.⁴ If an agent invests, she must buy a single unit of a critical input.⁵ An agent's output if she invests is the product of an exogenous *fundamental* $Z > 0$ and an endogenous *spillover*, where the latter is a strictly positive, strictly increasing function ψ of the proportion $\ell \in [0, 1]$ of agents who invest. An agent's payoff from investing is $Z\psi(\ell) - P$, where P is the input price. The assumption that output is multiplicatively separability in the fundamental and the spillover, while not essential for our results, considerably simplifies the model and its intuitions.

We assume that the fundamental $Z \sim G$ has a finite expected value and a continuous density whose support is the positive real line. In addition, the hazard function of the *log* fundamental, $\ln Z$, is strictly increasing, bounded on finite intervals, and goes to infinity as the fundamental does.⁶ Moreover, for any constant k , the conditional expectation $E(Z|Z > k)$ exists and is finite. Finally, the limit as $z \rightarrow \infty$ of $G'(z)z^2$ exists. (It may not be finite).

We assume the input is produced at a constant marginal cost $c \geq 0$. We contrast two mechanisms for the determination of the input price.

Monopoly case (M). A monopoly supplier chooses a price P to maximize its expected profits $(P - c)\ell$.⁷

Competitive case (C). A continuum of firms can each produce an unlimited quantity at the constant marginal cost c .

The timing is as follows. The input price P is announced and becomes common knowledge. In the competitive case, this price is just the marginal cost c . In the monopoly case, P is chosen to maximize expected supplier profits. Each agent i then sees a private signal $x_i = \ln Z + \tau \varepsilon^i$ of the log fundamental, where $\tau > 0$ is a scale factor and ε_i is idiosyncratic noise with a continuous density and support contained in $[-\frac{1}{2}, \frac{1}{2}]$.⁸ The noise terms ε_i and ε_j of any two distinct agents i and j are independent of each other and of Z . On seeing their signals, the agents simultaneously decide whether or not to invest. An agent's strategy is thus a function from her signal and the input price to a probability of investing. Once these investment decisions are made, the fundamental Z is revealed and all players receive their payoffs.

With a positive marginal cost c , the competitive case is a standard global game. However, the monopoly case is not, as the seller lacks dominance regions.

2.1. Equilibrium

We first solve the subgame played among the agents for a given input price P . We then use this result to derive expressions for the suppliers' profits and agent and social welfare. Finally, we solve for the equilibrium input price in our two cases and in the

⁴ The continuum assumption is made to save slightly on notation. The results are essentially the same with a finite number (at least two) of agents.

⁵ If the model is interpreted literally, the input may represent, e.g., oil. Alternatively, one can view the agents as consumers who consume a durable good (the "input") that displays network externalities, as in Argenziano (2008).

⁶ This assumption holds for the exponential, Laplace, and uniform distributions, as well as the gamma and Weibull with degrees of freedom parameter larger than one (Barlow and Proschan, 1975, p. 79). It also holds in a class of generalized normal distributions (Nadarajah, 2005).

⁷ If instead the monopolist has an increasing total cost function $C(\ell)$, our results still hold if one replaces the marginal cost c by the increase in the supplier's costs if all agents invest, $C(1) - C(0)$.

⁸ If agents received signals of the fundamental Z rather than its logarithm, and their signal errors had bounded support, our results would still hold. However, our proofs would be more lengthy as we would no longer be able to rely on a result of Morris and Shin (2000a, Proposition 2.2). In addition, technical problems would arise if the support of agents' signal errors were unbounded.

social optimum. Throughout, we focus on the limit as agents' signals become arbitrarily precise: $\tau \rightarrow 0$.

2.1.1. Subgame among agents

Proposition 1 characterizes the unique equilibrium of the subgame played by the agents and proves a useful result about the monopolist's optimal price. Let $\Psi = \int_{\ell=0}^1 \psi(\ell) d\ell$ denote the mean spillover.

Proposition 1. Fix an input price P . In the limit as the signal errors become small ($\tau \rightarrow 0$), all agents invest if the fundamental Z exceeds the threshold

$$\kappa = P/\Psi \tag{1}$$

and none invest if $Z < \kappa$. Moreover, a monopoly supplier will always choose a price $P \in (c, \infty)$.

Proposition 1 shows that, in the limit as the signal errors shrink, no agents invest if the fundamental Z is less than the threshold κ . We refer to this event as a "crisis"; it occurs with probability $\pi = G(\kappa)$. This result fully specifies the equilibrium of the model in the competitive case: since $P = c$, a crisis occurs with exogenous probability $G(c/\Psi)$. In the monopoly case, we must still solve for P .

2.1.2. Discussion: strategic rents

By Proposition 1, agents buy if and only if $P \leq \Psi Z$: their willingness to pay for the input is ΨZ . However, with vanishing noise, either all agents invest or none do. Thus, if an agent invests, her actual payoff is $\psi(1)Z - P$: an agent's valuation of the input, $\psi(1)Z$, strictly exceeds her willingness to pay, ΨZ . We call this gap the agent's *strategic rent*. In addition to their strategic rents $(\psi(1) - \Psi)Z$, the agents also receive their usual informational rents $\Psi Z - P$.

The existence of strategic rents is not due to risk aversion: the agents in our model are risk neutral. Rather, it is due to the effect of miscoordination costs on which equilibrium is selected. More precisely, consider a version of this game in which the agents see the fundamental Z without noise. For fundamentals Z that lie between $\frac{P}{\psi(1)}$ and $\frac{P}{\psi(0)}$, the cost P of investing lies between the benefit $\psi(0)Z$ of investing if none invest and the benefit $\psi(1)Z$ of investing if all invest. Hence, there are two pure Nash equilibria: all invest and none invest. The former is Pareto dominant since agents' payoffs in this equilibrium, $\psi(1)Z - P$, are positive, while they get nothing in the no-invest equilibrium.

If the agents could somehow coordinate on the all-invest equilibrium, their willingness to pay for the input would be their full-investment output $\psi(1)Z$. However, with noisy signals of the fundamental, they cannot coordinate. Rather, they choose the action that is a best response when the proportion ℓ of players who invest takes all values in $[0, 1]$ with equal probabilities (Morris and Shin, 2000a, Proposition 2.2). Under these beliefs, an agent's willingness to pay for the input is only ΨZ . However, when $P < \Psi Z$, all agents invest, so they produce their full-employment output $\psi(1)Z$. This exceeds their willingness to pay: the agents enjoy strategic rents.

2.2. Payoffs and welfare

The payoffs of the various participants are as follows. The expected profit Π of the monopolist is $\Pi = (P - c)(1 - \pi)$: the margin $P - c$ if a crisis is averted, times the probability $1 - \pi$ of this happening. By (1), these profits can be written

$$\Pi = \Psi \cdot \left(\kappa - \frac{c}{\Psi} \right) \cdot (1 - \pi). \tag{2}$$

Since the crisis risk π equals the probability $G(\kappa)$ that the fundamental falls below the investment threshold κ , we can

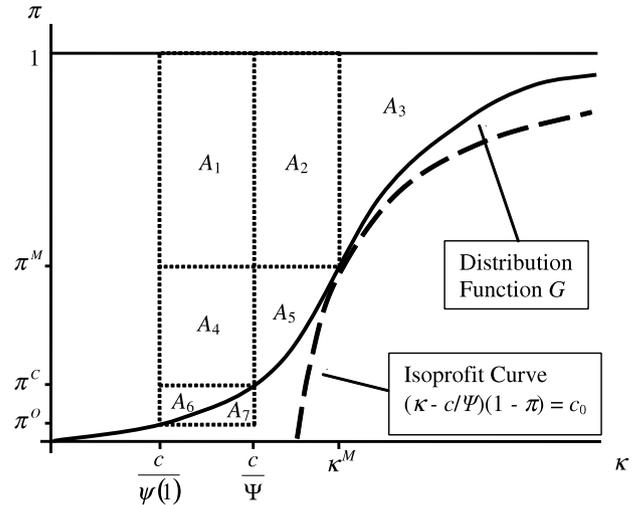


Fig. 1. Equilibrium, welfare, and social optimum. The investment threshold κ appears in the horizontal axis while the crisis probability π appears on the vertical axis. The solid curve is the distribution function $\pi = G(\kappa)$. The superscripts C, M, and O denote the competitive and monopoly cases, and the social optimum, respectively. The dashed curve is the supplier's profit-maximizing isoprofit curve for the monopoly case. The investment threshold κ in the competitive case and the social optimum equals c/Ψ and $c/\psi(1)$, respectively; in the monopoly case it is κ^M .

assume that the monopolist chooses κ directly rather than the price P .

We now fix an input price P and corresponding investment threshold $\kappa = P/\Psi$, and derive expressions for agent welfare and social welfare SW . The latter is the sum of agent welfare AW and supplier profits Π .

The realized payoff of the agents equals zero if the fundamental Z is less than the threshold κ . Else it equals their output $\psi(1)Z$ minus the input price $P = \Psi\kappa$. Agent welfare thus equals $\int_{z=\kappa}^{\infty} (z\psi(1) - \Psi\kappa) dG(z)$, which can be decomposed as follows:

$$AW = [\psi(1) - \Psi] \int_{z=\kappa}^{\infty} z dG(z) + \Psi \int_{z=\kappa}^{\infty} (z - \kappa) dG(z). \tag{3}$$

The first term is the expected difference between the agents' actual valuation, $\psi(1)Z$, and their reservation price, ΨZ . These are the agents' strategic rents: the rents they must receive in order to overcome their investment coordination problem. The second term, the expected difference between the agents' actual reservation price ΨZ and the monopolist's price $\Psi\kappa$, represents the usual informational rents that buyers receive by virtue of having private information about their willingness to pay.

The realized surplus equals output $\psi(1)Z$ less the input production cost c if the fundamental Z exceeds the threshold κ . Otherwise it is zero. Accordingly, social welfare is

$$\begin{aligned} SW &= \psi(1) \int_{z=\kappa}^{\infty} \left(z - \frac{c}{\psi(1)} \right) dG(z) \\ &= \psi(1) \int_{p=\pi}^1 \left(G^{-1}(p) - \frac{c}{\psi(1)} \right) dp, \end{aligned} \tag{4}$$

using the change of variables $p = G(z)$.

2.3. A geometric depiction

Building on the preceding results, the model has a simple geometric representation, depicted in Fig. 1. The agents' investment threshold κ appears in the horizontal axis while the crisis probability π appears on the vertical axis. The relationship between these two variables is given by the solid curve $\pi = G(\kappa)$. Superscripts

“M”, “C”, and “O” refer to the monopoly and competitive cases and the social optimum, respectively. Since the spillover function ψ is increasing, the full-investment spillover $\psi(1)$ exceeds the mean spillover Ψ , so $\frac{c}{\psi(1)}$ is less than $\frac{c}{\Psi}$ as indicated on the horizontal axis.

2.3.1. *Competitive case*

In the competitive case, the price $P = \Psi\kappa$ equals marginal cost c , so the equilibrium threshold κ equals c/Ψ . The resulting crisis risk, π^C , is depicted on the vertical axis. By (2), the input suppliers’ profits are zero; hence, agent and social welfare are identical. By (4), they equal the product of $\psi(1)$ and the sum of areas A_1 through A_5 .

2.3.2. *Social optimum*

By (4), social welfare is maximized at the investment threshold $\kappa = \frac{c}{\psi(1)}$, with a crisis risk $\pi^O = G\left(\frac{c}{\psi(1)}\right)$, as shown. Social welfare equals the full-investment spillover of $\psi(1)$ times the sum of areas A_1 through A_6 . Note that the efficient input price is $\Psi\kappa = c\frac{\Psi}{\psi(1)}$. This is less than the cost c of producing the input since the mean spillover Ψ is less than the full-investment spillover $\psi(1)$. Intuitively, investing creates positive spillovers by raising other agents’ profits. Hence, for any given input price P , the agents’ investment threshold is inefficiently high: it is P/Ψ , which exceeds the efficient investment threshold, $P/\psi(1)$. An input price that is below marginal cost is needed to offset this inefficiency, and operates like a subsidy to investment. As a result, input suppliers’ profits are negative in the social optimum: by (2), they lose an amount equal to the product of the mean spillover Ψ and the area of the rectangle that consists of areas A_1, A_4, A_6 , and A_7 . Simple accounting implies that agent welfare equals social welfare plus the absolute value of these losses.

2.3.3. *Monopoly case*

In the monopoly case, supplier profits Π are positive only to the right of the vertical line $\kappa = \frac{c}{\Psi}$, since only in this region does the input price $P = \Psi\kappa$ exceed the marginal cost c . The supplier will also pick a point below the horizontal line $\pi = 1$, since otherwise a crisis is certain, so profits are zero. Within the resulting region, a movement to the right corresponds to a higher input price and thus, since the crisis risk is held constant, higher profits. A movement downwards corresponds to a lower crisis probability and, since the input price is held fixed, higher profits. Hence, the monopolist’s isoprofit curves are upwards sloping.

The supplier seeks to choose its lowest isoprofit curve that touches the distribution function (the solid curve). This tangent isoprofit curve is shown, with point of tangency (κ^M, π^M) . Expected supplier profits Π equal the mean spillover Ψ times the area, $(\kappa^M - \frac{c}{\Psi})(1 - \pi^M)$, of the rectangle labeled A_2 . Social welfare equals the product of the full-investment spillover $\psi(1)$ and the sum of areas A_1, A_2 , and A_3 . Agent welfare simply equals the difference between social welfare and expected supplier profits.

The graphical approach of Fig. 1 departs from the usual textbook approach to monopoly pricing, which uses marginal revenue and marginal cost curves and is based on Cournot (1897, p. 56 ff.). The reason is that in our context, the analogue of the marginal revenue curve is derived from the distribution function in a complicated and unintuitive way.⁹ Our graphical approach, in contrast, relies

only on this distribution function and on the isoprofit curves, where the latter have a fixed parabolic shape.

The above analysis assumes that the monopolist has a unique interior optimum input price. More precisely, we have assumed that there exists a unique point of tangency between the distribution function G and the supplier’s isoprofit curves, at which profits are maximized rather than minimized. A sufficient condition for this is that as we move up the distribution function, the ratio of its slope to the slope of the intersecting isoprofit curve is strictly increasing, is less than one for low enough κ , and is greater than one for high enough κ . This guarantees that there is a unique point at which these slopes are equal, and that profits monotonically rise until we reach this point and monotonically fall as we move beyond it. The slope of the distribution function is $G'(\kappa)$ while the slope of the isoprofit curve is $\frac{1-\pi}{\kappa - \frac{c}{\Psi}}$,¹⁰ so their

ratio is $g(\kappa) \left(\kappa - \frac{c}{\Psi}\right)$ where $g(\kappa)$ is the hazard function $\frac{G'(\kappa)}{1-G(\kappa)}$. As the supplier will always choose a threshold $\kappa > \frac{c}{\Psi}$, it suffices that $g(\kappa) \left(\kappa - \frac{c}{\Psi}\right)$ be strictly increasing in κ when $\kappa > \frac{c}{\Psi}$, $\lim_{\kappa \downarrow \frac{c}{\Psi}} g(\kappa) \left(\kappa - \frac{c}{\Psi}\right) = 0$, and $\lim_{\kappa \rightarrow \infty} g(\kappa) \left(\kappa - \frac{c}{\Psi}\right) = \infty$. These properties hold by the following claim:

Claim 1. For any constant $a \geq 0$, the function $g(z) (z - a)$ is continuous and strictly increasing on $z \in [a, \infty)$ and satisfies $\lim_{z \downarrow a} g(z) (z - a) = 0$ and $\lim_{z \rightarrow \infty} g(z) (z - a) = \infty$.

2.4. *Comparative statics*

This section derives two comparative statics results. In Section 2.4.1 we study long-run growth in fundamentals when shocks are log-stationary. This growth shrinks the crisis risk to zero in the competitive case but not under monopoly. In this sense, monopoly rent extraction can leave the economy perpetually vulnerable to crises.

In Section 2.4.2, we assume that the supplier sees a public signal of fundamentals before choosing its price. An increase in the precision of this signal leads to a lower crisis risk and raises social welfare and the monopolist’s profits. The agents benefit from the lower crisis risk but lose informational rents to the monopolist, so agent welfare may rise or fall. In the limit as the public signal becomes arbitrarily precise, the agents have no informational rents but still have positive strategic rents. This contrasts with the standard monopoly case without strategic complementarities, in which only informational rents are present.

2.4.1. *Recurrent crises*

In a dynamic setting, the distribution of the fundamental is likely to fluctuate. For instance, technical progress will tend to raise agents’ productivity over time. To study this phenomenon, we take a parametric approach. Letting μ and σ^2 , respectively, be the mean and variance (assumed finite) of the log fundamental $\ln Z$, we can always write $\ln Z = \mu + \sigma u$, where u is a random variable with mean zero and unit variance. We now study the effects of varying the mean μ of the log fundamental while holding the variance σ^2 and the distribution H of u fixed.

We find that unbounded growth in the expected log fundamental μ drives the crisis risk to zero in the competitive case, but not in the monopoly case.

marginal revenue curve in (Q, P) space is given by the complicated expression $P = \left(G^{-1}(1-Q) - \frac{Q}{G(G^{-1}(1-Q))}\right) \Psi$. (This can be shown by differentiating revenue $\Psi\kappa(1-\pi)$ with respect to $1-\pi$, obtaining $\left(\kappa - \frac{1-\pi}{G'(\kappa)}\right) \Psi$, and then using the definition of Q and $\kappa = G^{-1}(\pi)$ to rewriting this expression in terms of Q alone.) This approach appears, to us, to be much less intuitive than the one we adopt.

¹⁰ This is computed as $-\frac{\partial[(\kappa - c/\Psi)(1-\pi)]/\partial\kappa}{\partial[(\kappa - c/\Psi)(1-\pi)]/\partial\pi}$.

⁹ The traditional approach would involve depicting the expected quantity sold, $Q = 1 - \pi$, on the horizontal axis and the price, $P = \Psi\kappa$, on the vertical axis. These variables are related by the equation $Q = 1 - G(P/\Psi)$, which is analogous to the demand curve in the classical analysis. The graph of the

Theorem 1. Suppose $\ln Z = \mu + \sigma u$, where μ and $\sigma > 0$ are constants and u has the fixed distribution H with zero mean, unit variance, and a strictly increasing and continuous hazard function $h(u) = \frac{H'(u)}{1-H(u)}$ whose range includes an open neighborhood of σ .

1. In the competitive case, as either the mean $\mu \rightarrow \infty$ or the cost $c \rightarrow 0$, agent welfare (which equals social welfare) continually rises while the crisis risk π falls monotonically to zero.
2. In the monopoly case, as either $\mu \rightarrow \infty$ or $c \rightarrow 0$,
 - (a) the supplier's profits and agent and social welfare rise monotonically;
 - (b) The crisis risk π falls monotonically to the strictly positive limit $H(h^{-1}(\sigma))$ (which shrinks to zero as σ does);
 - (c) The input price P rises (falls) as μ rises (c falls), but in both limits, $e^{-\mu}P$ falls monotonically to the constant $\Psi e^{\sigma h^{-1}(\sigma)}$.

Moreover, there is continuity in the limit: the limits, as either $\mu \rightarrow \infty$ or $c \rightarrow 0$, of π and $e^{-\mu}P$ are each equal to the values these quantities take when $c = 0$.

In the competitive case (part 1 of the theorem), the investment threshold is fixed. With a positive trend in fundamentals, the chance that the fundamental lies below this fixed threshold shrinks to zero: crises disappear. In contrast, a monopoly supplier responds to growth in the mean log fundamental μ by raising the input price. Hence, while the crisis risk falls, it is bounded below by a strictly positive constant (parts 2(b) and 2(c)). Intuitively, an increase in μ raises both the mean and the variance of the fundamental $Z = e^{\mu + \sigma u}$. Hence, the supplier's incentive to cream-skim – to forego sales in bad states in return for a higher price in good states – does not vanish. Consistent with this intuition, the limiting crisis risk $H(h^{-1}(\sigma))$ is an increasing function of the standard deviation σ of the shocks to the log fundamental, and goes to zero as σ does (part 2(b)).

A dynamic version. While our model is static, it has implications for how an economy with rent extraction behaves in the long run. This can be made precise by embedding the model in a dynamic setting, as follows. A unit measure of agents $i \in [0, 1]$ interact in each of an infinite sequence of periods $t = 1, 2, \dots$ ¹¹ In each period t , the fundamental Z_{t-1} from the prior period is commonly observed. Each player i then sees a private signal $x_t^i = \ln Z_t + \tau \varepsilon_t^i$ of the contemporaneous log fundamental. Moreover, the fundamental follows the stochastic process $\ln Z_t = \ln Z_{t-1} + m + \sigma u_t$, where m , σ , and Z_0 are positive constants and the shock u_t has a stationary distribution H , whose hazard function is strictly increasing, bounded on finite intervals, and goes to infinity as u_t does. An agent's payoff from investing in period t is $Z_t \psi(\ell_t) - P_t$, where ℓ_t is the proportion of agents who invest in period t and P_t is the input price. The input is supplied either competitively or by a monopolist. In either case, a supplier's constant marginal cost is c , and suppliers maximize profit period by period. Since history matters only through the prior fundamental variable Z_{t-1} which is commonly observed, play in each period t is strategically equivalent to our static model if we associate Z_t with Z and $\ln Z_{t-1} + m$ with μ . Hence, Theorem 1's predictions of what happens as the mean μ of $\ln Z$ goes to infinity hold also for this dynamic model in the long run as the fundamental Z_{t-1} grows without bound.

2.4.2. Public information

What happens if, prior to setting its price, the supplier observes a public signal of the fundamental? In this section, we modify our

model in order to study this question. For tractability, we now assume that the log fundamental $\ln Z$ is normally distributed with mean μ and variance σ^2 . Initially, all players see a public signal Y , equal to the log fundamental plus an error term: $Y = \ln Z + \epsilon$. The public signal error ϵ is also normal, with mean zero and variance s^2 . The rest of the model proceeds as before: the input price is announced, private signals are observed, and agents make their investment decisions. The public signal error ϵ is independent of the private signal errors.

We focus on the case of zero input production costs ($c = 0$) in the limit as the private signals become arbitrarily precise.¹² In this limit, agents' actions are independent of the public signal. Hence, the public signal has no effect on the outcome in the competitive case, since the input is sold at cost. The remainder of this section will study the effects in the monopoly case.

Our main result is that, even as the public signal becomes arbitrarily precise, agent welfare remains bounded below by a strictly positive constant.¹³ In contrast, in the standard monopoly case without strategic complementarities (when $\Psi = \psi(1)$), agent welfare converges to zero. Intuitively, the best the supplier can do is to charge a price slightly below the investors' willingness to pay, ΨZ . However, even with such a price, the investors receive strictly positive strategic rents $[\psi(1) - \Psi]Z$.

This result appears formally in the following theorem. We also show that a more precise public signal lowers the crisis risk, raises social welfare, and helps the monopolist.

Theorem 2. Assume the input production cost c is identically zero.

1. Both ex ante social welfare SW and the supplier's ex ante expected profits are strictly increasing in the precision of the public signal, while the crisis risk is decreasing in this precision.
2. As the precision of the public signal grows without bound, agent welfare AW converges to the strictly positive constant $[\psi(1) - \Psi]E(Z)$.

An intuition for why the supplier prefers a more precise public signal is due to Blackwell (1953). Since a less precise signal equals a more precise signal plus independent noise, the supplier cannot do better with a less precise signal, as the additional noise prevents it from best-responding to the more precise signal.¹⁴

Outside of the limit, a more precise public signal may help or hurt the agents. This is shown in a computed example in Fig. 2. The standard deviation s of the public signal error is shown on the horizontal axis. As implied by Theorem 2, a more precise public signal raises both social welfare and the monopolist's profits and lowers the crisis risk. As for the agents, there are two effects. First, they have smaller informational rents. This hurts them. However, the monopolist's better ability to target their willingness to pay allows it to avoid a crisis more often. This helps the agents. The net effect on agent welfare depends on which effect dominates.

3. Ruling out other solutions

We have shown that one can obtain recurring crises in a global game by relaxing the assumption of dominance regions. Alternatively, one can make assumptions on the fundamental that guarantee that it will continually jump back and forth over the investors' crisis threshold.

¹¹ As the agents are anonymous and cannot save, their optimal behavior is myopic. Hence, it does not matter whether the agents in each period are the same or an infinite sequence of distinct generations.

¹² We adopt the standard convention in which the precision of the public signal refers to the reciprocal, $1/s^2$, of its variance.

¹³ These results assume that the variance of the private signals approaches zero faster than the variance of the public signal. If this is not done, multiple equilibria could re-emerge (e.g., Morris and Shin, 2000a).

¹⁴ This might not hold in a dynamic context in which agents will invest more now if they expect the supplier to extract less rent from them in the future. We do not explore this issue in the present paper.

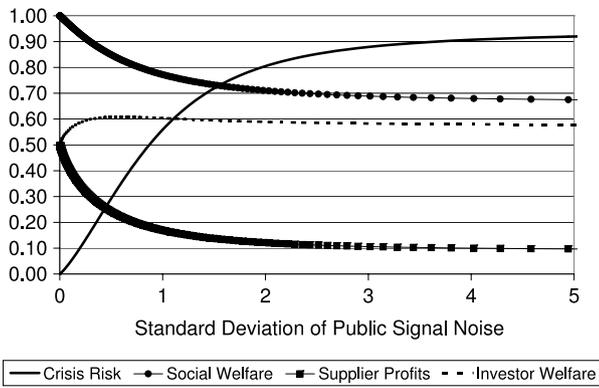


Fig. 2. Noisy public signal: computed example, monopoly supplier. This example assumes that the full-investment spillover is twice the mean spillover ($\psi(1) = 2\psi$) and the standard deviation σ of the log fundamentals equals 2. The standard deviation s of the noise in the public signal appears on the horizontal axis. Payoffs are normalized to reach a maximum value of one.

We now show, in a precise sense, that no other approach will work. We assume a continuum of players who play a normal form stage game in each of an infinite number of periods.¹⁵ A player's action is her strategy in this stage game. Players are anonymous: only the aggregate action distribution is observed. Each agent's payoff in the stage game depends on a vector of exogenous private signals, and possibly also on a vector of unobserved, common fundamental variables.

A player's signals can play several roles. They can be independent across players, in which case a player's signal can be interpreted as her "type". They may be imperfectly correlated, as in the case of a global game. Or they may be perfectly correlated, as in the case of a symmetric-information game. In addition, some of the signals may not be directly payoff-relevant (although at least one of them must, for there to be dominance regions). If these payoff-irrelevant signals are perfectly correlated across players, they can play the role of a sunspot (Cass and Shell, 1983). If they are imperfectly correlated, they can play the role of a coordinating device, as in a correlated equilibrium (Aumann, 1974).

We make three key assumptions. First, we assume that the value of one signal is asymptotically unpredictable given the contemporaneous values of the other signals.¹⁶ Second, arbitrarily large reversals in this signal – downwards (upwards) jumps when the signal is high (low) – are unlikely. Finally, we assume dominance regions: when the given signal is sufficiently high or low, each player has an action that is strictly dominant in the stage game. The location of a player's dominance regions may depend in an arbitrary (but Lipschitz-continuous) way on her other signals. The assumption of dominance regions means that our results will be mainly relevant to global games.

Under these assumptions, we show that players eventually spend arbitrarily long stretches of time in one or the other of their dominance regions. This implies that play must then converge in the long run, in the sense that the unconditional (time-zero) expectation of the measure of players who change their actions from one period to the next converges to zero. Hence, situations (such as crises and fluctuations) in which a large proportion of agents change their actions become increasingly rare.¹⁷ In Appendix A,

¹⁵ There are no endogenous state variables, such as capital, and no learning; hence, it does not matter whether the same or different sets of players interact over time.

¹⁶ This is the assumption that is violated in the models of Chamley (1999), Oyama (2004), Burdzy and Frankel (2005), and Frankel and Pauzner (2000).

¹⁷ We also consider the case in which the original signal vector does not satisfy the three assumptions (e.g., because it is cointegrated), but there exists an invertible transformation of this signal vector that does. In this case, fluctuations also vanish (Corollary 1).

we show that our assumptions are jointly minimal: if any of them is weakened unilaterally, then play may not stabilize in the long run.¹⁸

The model is as follows. There is a unit measure of players $i \in [0, 1]$ who play a simultaneous-move stage game in each of an infinite number of periods $t = 0, 1, \dots$. Let $a_t^i \in A^i$ be the action chosen by player i in period t . Let a_t^{-i} denote the action profile of player i 's opponents and let A^{-i} be the set of all such action profiles.¹⁹

A player's payoff in the stage game is given by a function u_i of her action a_t^i , the opposing action profile a_t^{-i} , a private signal vector $X_{it} \in \mathfrak{N}^k$, and a vector of unobserved fundamentals $B_t \in \mathfrak{N}^l$ that are common to the players. Since payoffs can depend directly on signals, unobserved fundamentals, or both, our model includes both the private-values and common-values global games as special cases (Morris and Shin, 2000a, pp. 64–68). In particular, there are private values if $L = 0$, while there are common values if $L \geq 1$ and the signal vector X_{it} has no direct effect on payoffs.

Let X denote the sequence of players' signal vectors: $X = ((X_{it})_{i \in [0,1]})_{t=0}^\infty$. Let $B = (B_t)_{t=0}^\infty$ be the sequence of unobserved fundamental vectors. The pair $\omega = (X, B)$ can be interpreted as the state of the world. Let Ω be the set of all such states.

For any vector $V = (v_1, \dots, v_M)$, let $V^{-m} = (v_1, \dots, v_{m-1}, v_{m+1}, \dots, v_M)$ denote the vector that results from removing the m th component of V . We will often need to refer to the proportion of players in period t who have signal vectors X_{it} that satisfy some weak inequality, such as $X_{it}^1 \leq h(X_{it}^{-1})$ for some continuous function h . The set of signal vectors that satisfies such an inequality is closed. Thus, we will be interested in the proportion of players in period t whose signal vector X_{it} lies in some closed set Σ_i . Sometimes, the inequality will involve values of the signal vector in two adjacent periods, t and $t + 1$; e.g., the condition $\max_k |X_{it}^k - X_{i,t+1}^k| \geq \delta$. The set of pairs $(X_{it}, X_{i,t+1})$ of adjacent signal vectors that satisfy such an inequality is a closed subset of \mathfrak{N}^{2k} .

To handle these quantities, we assume the existence of probability functions P_{t_1, \dots, t_N}^i for each player i and sequence t_1, \dots, t_N of distinct periods such that for any assignment of closed sets $\Sigma_i \in \mathfrak{N}^{NK}$ to players i , the integral

$$\int_{i=0}^1 P_{t_1, \dots, t_N}^i (X_{it_1} \times \dots \times X_{it_N} \in \Sigma_i) di$$

exists and equals the expected proportion of players i for whom the Cartesian product of signal vectors in periods t_1, \dots, t_N lies in the set Σ_i . If the set Σ_i is a Cartesian product of closed sets Σ_{it_n} , one for each period t_n , we can write the integrand more simply as $P_{t_1, \dots, t_N}^i (X_{it_1} \in \Sigma_{it_1}, \dots, X_{it_N} \in \Sigma_{it_N})$. This can be interpreted as the probability that in each period t_n , player i 's signal vector X_{it_n} lies in the set Σ_{it_n} . Finally, we will often substitute the actual inequality that X_{it} must satisfy; for instance, $\int_{i=0}^1 P_t^i (X_{it}^1 \leq h(X_{it}^{-1})) di$ is the proportion of players i in period t for whom $X_{it}^1 \leq h(X_{it}^{-1})$.

We now state our three assumptions. The first is that one signal – which without loss of generality we take to be the first component of the signal vector – is asymptotically unpredictable conditional on the other signals. Let F be the set of real-valued functions $f(i, X_{it}^{-1})$ that are Lipschitz-continuous in the signal vector X_{it}^{-1} .²⁰ The assumption is as follows.

¹⁸ This result assumes private values: a player's payoff depends directly on her signals, not on fundamentals. We also prove a weaker version of joint minimality in the general case.

¹⁹ More precisely, the opposing action profile a_t^{-i} is a function from opposing players to actions, where $a_t^{-i}(j) \in A^j$ is the action of player $j \neq i$. A^{-i} is the set of all such functions.

²⁰ The Lipschitz constant of $f(i, \cdot)$ may vary across players i and need not be a bounded function of i .

Asymptotic unpredictability. For any positive interval length $c < \infty$, probability $\varepsilon > 0$, and function $f \in F$, there is a $t^* < \infty$ such that at all times t after t^* , the expected measure of players whose first signal lies in an interval of length c with left endpoint equal to $f(i, X_{it}^{-1})$ is less than ε : for all $t > t^*$,

$$\int_{i=0}^1 P_t^i (X_{it}^1 \in [f(i, X_{it}^{-1}), f(i, X_{it}^{-1}) + c]) di < \varepsilon.$$

That is, even if one optimally uses the information from the player's identity and the player's other signals about where (in which fixed-length interval) the first signal is located, the ex ante probability that one will guess correctly goes to zero in the long run.²¹

Asymptotic Unpredictability allows the change in a player's first signal to depend on the changes in her other signals, as long as this change has some small degree of unpredictability that accumulates over time. For instance, suppose the increment in the first signal equals the increment in the second signal plus some small i.i.d. noise: $\Delta X_{it}^1 = \Delta X_{it}^2 + \varepsilon_t$ where $\sigma > 0$ is arbitrarily small and $\varepsilon_t \sim N(0, \sigma^2)$. (For any time series $(z_t)_{t=0}^\infty$, we let Δz_t denote $z_t - z_{t-1}$.) Then conditional on the second signal, the first signal has a normal distribution with variance $t\sigma^2$, so the property holds.

Next, we assume that a particular type of large change in the signal vectors is asymptotically rare. For any $\lambda > 0$, let F^λ be the subset of F consisting of functions f whose Lipschitz constant for each player is at most λ . For any function $f \in F^\lambda$, we assume that the long-run probability that $X_{it}^1 - f(i, X_{it}^{-1})$ and $X_{i,t+1}^1 - f(i, X_{i,t+1}^{-1})$ have opposite signs and each has an absolute value of at least δ goes to zero as $\delta \rightarrow \infty$:

Large reversal rarity. For any Lipschitz constant $\lambda > 0$ and probability $\varepsilon > 0$, there is a magnitude $\delta < \infty$ such that for all functions f in F^λ , there is a time $t^* < \infty$ such that at all times $t > t^*$, the ex ante expected proportion of players for whom $X_{it}^1 - f(i, X_{it}^{-1})$ and $X_{i,t+1}^1 - f(i, X_{i,t+1}^{-1})$ have opposite signs and each has an absolute value of at least δ is less than ε :

$$\begin{aligned} & \int_{i=0}^1 P_{t,t+1}^i (X_{it}^1 \leq f(i, X_{it}^{-1}) \\ & \quad - \delta, X_{i,t+1}^1 \geq f(i, X_{i,t+1}^{-1}) + \delta) di + \int_{i=0}^1 P_{t,t+1}^i (X_{it}^1 \\ & \quad \geq f(i, X_{it}^{-1}) + \delta, X_{i,t+1}^1 \leq f(i, X_{i,t+1}^{-1}) - \delta) di < \varepsilon. \end{aligned}$$

Note that δ and t^* can depend on the initial values X_{i0} of the signal vector, as they might have to in the case of a mean-reverting process (e.g., if $E_{t-1} \Delta X_{it}^k = -X_{i,t-1}^k$ for some signal k other than the first). This assumption is clearly very weak. For instance, it is implied by the following property:

Large jump rarity. In the long run, the proportion of players who have arbitrarily large jumps in any signal goes to zero: for all $\varepsilon > 0$ there is a $\delta < \infty$ and a $t^* < \infty$ such that at all times t after t^* ,

$$\int_{i=0}^1 P_{t,t+1}^i \left(\max_k |X_{i,t+1}^k - X_{it}^k| \geq \delta \right) di < \varepsilon.$$

Claim 2. Large Jump Rarity implies Large Reversal Rarity.

²¹ Time itself is not an argument of the function f . Hence, Asymptotic Unpredictability states merely that in the long run, the first signal is unlikely to lie in a fixed-length interval whose location is any time-invariant function of the other signals. The location of the first signal may still be predictable as a function of time. One example is an AR(1) process with a deterministic time trend.

Our last assumption is that, for any player i , there are two actions \bar{a}_i and \underline{a}_i such that for any vector of signals 2 through K , \bar{a}_i is strictly dominant in the stage game when signal 1 is high enough, and \underline{a}_i is strictly dominant when signal 1 is low enough. In addition, for each player, the set of values of the first signal that lie between these two "Dominance Regions" lies within a fixed-length interval whose location depends in a Lipschitz-continuous way on the other signals. For any vector $x \in \mathfrak{N}^K$, player i , and period t , let $B(x, i, t)$ denote the set of fundamentals $B_t \in \mathfrak{N}^L$ that occur with positive probability if player i 's signal vector at time t , X_{it} , equals x .²²

Dominance regions. There exists a set $S \subseteq [0, 1]$ of players, whose complement has measure zero, together with actions \underline{a}_i and \bar{a}_i for each player i in S and functions $c_0 : [0, 1] \rightarrow [0, \infty)$ and $c \in F$ such that for each vector X_{it}^{-1} of signals 2 through K :

1. if $X_{it}^1 < c(i, X_{it}^{-1}) - c_0(i)$, then for any feasible fundamental $B_t \in B(X_{it}, i, t)$ and opposing action profile $a^{-i} \in A^{-i}$, every action $a^i \neq \underline{a}^i$ yields a lower payoff for player i than \underline{a}^i :

$$u_i(a^i, a^{-i}, X_{it}, B_t) < u_i(\underline{a}^i, a^{-i}, X_{it}, B_t)$$
2. if $X_{it}^1 > c(i, X_{it}^{-1})$, then for any feasible fundamental $B_t \in B(X_{it}, i, t)$ and opposing action profile $a^{-i} \in A^{-i}$, every action $a^i \neq \bar{a}^i$ yields a lower payoff for player i than \bar{a}^i :

$$u_i(a^i, a^{-i}, X_{it}, B_t) < u_i(\bar{a}^i, a^{-i}, X_{it}, B_t).$$

We will call the region above $c(i, X_{it}^{-1})$ player i 's upper dominance region and the region below $c(i, X_{it}^{-1}) - c_0(i)$ player i 's lower dominance region. The interval $[c(i, X_{it}^{-1}) - c_0(i), c(i, X_{it}^{-1})]$ will be called player i 's nondominance region. This interval contains all signals X_{it}^1 for which there is no dominant action given the other signals X_{it}^{-1} . Since it is of fixed length for each player, it may also contain a set of signals X_{it}^1 for which player i does have a dominant action given X_{it}^{-1} . That will turn out to be immaterial for our results.

We have not assumed that the action sets A^i are bounded. However, for the assumption of Dominance Regions to make sense in most contexts, \bar{a}_i and \underline{a}_i will generally correspond to highest and lowest actions. Hence, our results are most relevant when action spaces are finite, or at least compact.

For Dominance Regions to hold, the set $B(X_{it}, i, t)$ of feasible fundamentals B_t must depend nontrivially on the signal-vector X_{it} . This is so, for instance, if each player receives a noisy signal of fundamentals and the support of her signal error is bounded. This assumption is common in models of global games, starting with Carlsson and van Damme (1993). However, some authors have instead assumed unbounded signal errors (e.g., Morris and Shin, 2000b). This leads to complications in a repeated setting, since a player's beliefs about fundamentals depend not only on her own signal, but also on other players' past actions. For instance, a player who sees a high signal might not conclude that the fundamental is high if she has seen other players choose low actions in recent periods. Establishing convergence in this case would require additional restrictions on players' strategies and/or beliefs. We leave this question for future research.

Our notion of convergence is as follows.

Vanishing fluctuations. For any $\varepsilon > 0$, there is a $t^* < \infty$ such that for all $t > t^*$, the unconditional expected proportion

²² More precisely, $B(x, i, t)$ is the set of vectors $b \in \mathfrak{N}^L$ such that there is a positive-probability set of states $\omega \in \Omega$ for which $X_{it} = x$ and $B_t = b$.

of players i who are in the same dominance region in periods t and $t + 1$ (and who thus choose the same action in the two periods) is at least $1 - \varepsilon$:

$$\begin{aligned} & \int_{i=0}^1 P_{t,t+1}^i \left(X_{it}^1 \leq c(i, X_{it}^{-1}) - c_0(i), X_{i,t+1}^1 \right) \\ & \leq c(i, X_{i,t+1}^{-1}) - c_0(i) \, di + \int_{i=0}^1 P_{t,t+1}^i \left(X_{it}^1 \right) \\ & \geq c(i, X_{it}^{-1}), X_{i,t+1}^1 \geq c(i, X_{i,t+1}^{-1}) \, di \geq 1 - \varepsilon. \end{aligned}$$

If Vanishing Fluctuations holds, then aggregate fluctuations – large changes in the aggregate action distribution – disappear in the long run.

We now show that our three assumptions imply Vanishing Fluctuations. Note that we have assumed nothing about the strategic interactions between the players; in particular, there can be strategic complements or substitutes.

Theorem 3. *Asymptotic Unpredictability, Dominance Regions, and Large Reversal Rarity jointly imply Vanishing Fluctuations.*

The three assumptions of Theorem 3 apply to a particular component of players' signal vectors. This may seem restrictive. However, the following corollary shows that this is deceptive: there merely must be some invertible transformation of the signal vector, such that the first signal in the transformed vector satisfies our properties with respect to the corresponding payoff function. The proof, which is trivial, is omitted.

Corollary 1. *Suppose that the signal vector X_{it} of each player i lies in some set $D_i \subseteq \mathfrak{R}^K$. Let T_i be a one-to-one function from D_i into \mathfrak{R}^M for some $M \geq 1$. Define $\widehat{X}_{it} = T_i(X_{it})$ to be the transformed signal vector and $\widehat{u}_i(a_t^i, a_t^{-i}, \widehat{X}_{it}, B_t) = u_i(a_t^i, a_t^{-i}, T_i^{-1}(\widehat{X}_{it}), B_t)$ to be the associated payoff function. Let $\widehat{X} = \left(\widehat{X}_{it} \right)_{i \in [0,1], t=0}^\infty$ be the collection of transformed signal vectors and let \widehat{G} be the probability distribution over states (\widehat{X}, B) that is defined by $\widehat{G}((\widehat{X}, B)) = G((X, B))$. If \widehat{G} satisfies Asymptotic Unpredictability and Large Reversal Rarity and the payoff functions \widehat{u}_i satisfy Dominance Regions, then Vanishing Fluctuations holds.*

This corollary expands the empirical relevance of Theorem 3. For instance, suppose all players see the common signal vector $X_t = (X_t^1, X_t^2)$, where X_t^1 is the price of oil and X_t^2 is the price of electricity, both in logarithmic form. (Assume that there is no unobserved fundamental.) Subsequently, each player decides whether or not to invest. Assume, plausibly, that X_t^1 and X_t^2 are cointegrated (Granger, 1981); e.g., $X_t^1 = X_{t-1}^1 + \varepsilon_t$ and $X_t^2 = X_t^1 + \nu_t$, where ε_t and ν_t are white noise (standard normal). Thus, the oil price follows a random walk, and the electricity price in any period equals the contemporaneous oil price plus white noise. This violates Asymptotic Unpredictability, since the gap between the prices is standard normal in every period. Hence, Theorem 3 does not apply directly.

Now suppose we transform the signal vector as follows: $\widehat{X}_t = (X_t^1, X_t^2 - X_t^1)$. The first signal is still the oil price, but the second signal is now the gap between the two prices. Asymptotic Unpredictability now holds, since the transformed signals are independent and the unconditional variance of the first one grows without bound. Moreover, \widehat{X}_t clearly satisfies Large Jump Rarity and, thus, Large Reversal Rarity. Finally, suppose that for any given price gap, investing is a strictly dominant choice if oil (and hence electricity) is cheap enough, while not investing is strictly dominant if oil (and thus electricity) is sufficiently expensive. Then the corresponding payoff function stated in terms of \widehat{X}_t will satisfy

Dominance Regions. By Corollary 1, crises will vanish in the long run.

Theorem 3 follows from two lemmas. The first states that for any finite $b_1 > 0$, it eventually becomes arbitrarily likely that a player's first signal lies at least b_1 away from her nondominance region.

Lemma 1. *Assume Asymptotic Unpredictability and Dominance Regions. For any $b_1 \in (0, \infty)$ and any $\varepsilon_1 > 0$ there is a $t_1 < \infty$ such that if $t > t_1$, then the expected proportion of players i whose first signal X_{it}^1 lies within b_1 of their nondominance region is less than ε_1 :*

$$\int_{i=0}^1 P_t^i \left(X_{it}^1 \in [c(i, X_{it}^{-1}) - c_0(i) - b_1, c(i, X_{it}^{-1}) + b_1] \right) di < \varepsilon_1.$$

The next lemma states that in the long run, the expected proportion of players who are very far from their nondominance regions in one period and enter or skip over this region in the next period is very low. For any player i and constant $\kappa > 0$, let $\Sigma_i^{jump}(\kappa)$ be the set of pairs of signal vectors $(X_{it}, X_{i,t+1}) \in \mathfrak{R}^{2K}$ such that player i is a distance of at least κ from her nondominance region in period t and she enters or jumps over this region in period $t + 1$.²³

Lemma 2. *Assume Large Reversal Rarity and Dominance Regions. For any $\varepsilon_2 > 0$ there is a $t_2 < \infty$ and $b_2 \in (0, \infty)$ such that if $t > t_2$, then the expected proportion of players who are a distance of at least b_2 from their nondominance regions in period t and enter or skip over their nondominance regions in the next period is less than ε_2 :*

$$\int_{i=0}^1 P_{t,t+1}^i \left((X_{it}, X_{i,t+1}) \in \Sigma_i^{jump}(b_2) \right) di < \varepsilon_2.$$

Given these lemmas, the intuition for Theorem 3 is straightforward. By Lemma 1, the proportion of players who are close to their nondominance regions is very small in the long run. Moreover, by Lemma 2, the distance of a player from her nondominance region is unlikely to change by a very large amount in one period. Hence, the vast majority of players will eventually have the same dominant action in a given period as in the next period.

4. Concluding remarks

In the global games approach of Carlsson and van Damme (1993), players get conditionally independent signals of a random fundamental. For extreme values of these signals, players have strictly dominant actions. By a contagion argument, the equilibrium is unique and characterized by one or more thresholds at which aggregate behavior changes abruptly. These properties make global games useful for studying crises and aggregate fluctuations.

The uniqueness result relies on the assumption of dominance regions. This assumption becomes a weakness when the game is repeated. If the fundamental follows a random walk, it will eventually wander far into the dominance regions: crises will become increasingly rare. In response, researchers have made assumptions on the fundamental that guarantee that it will jump back and forth over the threshold(s) at which aggregate behavior shifts.

We propose an alternative: dropping the assumption of dominance regions for one large player. We study a continuum of agents who play a global game of investment, as in Morris and Shin (2000a, pp. 59 ff.). In order to invest, an agent must buy a critical

²³ This holds if either (a) $X_{it}^1 \geq c(i, X_{it}^{-1}) + \kappa$ and $X_{i,t+1}^1 \leq c(i, X_{i,t+1}^{-1})$ or (b) $X_{it}^1 \leq c(i, X_{it}^{-1}) - c_0(i) - \kappa$ and $X_{i,t+1}^1 \geq c(i, X_{i,t+1}^{-1}) - c_0(i)$.

input from a price-setting monopolist. After the monopolist sets its price, agents get slightly noisy signals of a random fundamental and decide whether or not to invest. Since there is no highest price, the monopolist lacks dominance regions.

Unlike in Morris and Shin (2000a, pp. 59 ff.), the agents' investment threshold is not constant: it is an increasing function of the input price. Consequently, the monopolist trades off the gain from higher prices against the greater crisis risk that results when the agents choose a higher threshold. Positive shocks cause the monopolist to raise its price, leaving the economy perpetually vulnerable to bad shocks. In a parametric setting with log-stationary shocks, the crisis risk converges to a strictly positive constant as the mean of the log fundamental goes to infinity: crises do not disappear in the long run.

We also show that in a parametric setting with a normally distributed public signal of a normal log-fundamental, a decrease in the standard deviation of this signal enables the supplier to extract rents more precisely. Investor welfare may fall despite the accompanying reduction in the crisis risk. However, in contrast with the standard monopoly model, investor welfare does not converge to zero as the public signal becomes arbitrarily precise. Intuitively, strategic complementarities in investment create strategic risk among investors. This lowers their willingness to pay for the investment good below the benefit they will get from this good if all invest. Hence, the investors receive strategic rents that do not vanish as the monopolist becomes more informed about the fundamental.

As noted, recurring crises can be obtained by forcing the fundamental to jump back and forth over the crisis threshold(s), or by relaxing dominance regions for at least one player so that the threshold can track the fundamental. We also show that these are the only two solutions, in the following precise sense. Consider a continuum of anonymous players who interact repeatedly over time. A player's payoff depends on a vector of exogenous, random private signals (e.g., of common shocks). Assume that each player has at least one signal that displays dominance regions, is asymptotically unpredictable, and does not jump back and forth by increasingly large amounts. Then the crisis frequency must shrink to zero in the long run.

Acknowledgments

I thank George-Marios Angeletos, Drew Fudenberg, Hans Carlsson, Guido Lorenzoni, Ivan Werning, and workshop participants at U. Minnesota, MIT, NYU, Washington U., Wharton, and the 2009 North American Meetings of the Econometric Society for helpful comments.

Appendix A. Joint minimality

In this appendix, we show that the three assumptions of Theorem 3 are jointly minimal. In the model of Section 3, let G be a distribution over states; each state consists of a sequence of fundamental vectors, as well as a sequence of signal vectors for each player.

Theorem 4. *Assuming private values (no unobserved fundamentals), the assumptions of Theorem 3 are jointly minimal:*

1. For any state distribution G that violates either Asymptotic Unpredictability or Large Reversal Rarity, there are payoffs that satisfy Dominance Regions, such that Vanishing Fluctuations does not hold.
2. For any payoffs that violate Dominance Regions, there is a state distribution G that satisfies Asymptotic Unpredictability and Large Reversal Rarity, such that Vanishing Fluctuations does not hold. Moreover, part 1 still holds with unobserved fundamentals.

The presence of unobserved fundamentals matters since they impose a consistency requirement on players' signal vectors: for two signal vectors to occur in the same period, they must both be consistent with a given realization of fundamentals. It is entirely possible that a positive-measure set of players i each has a vector X_{it}^{-1} of signals 2 through K for which the player lacks dominance regions in the first signal X_{it}^1 , but that there is no fundamental vector B_t for which any two of these signal vectors (i.e., X_{it}^{-1} and X_{jt}^{-1} for $i \neq j$) can occur simultaneously. On the other hand, this problem does not affect part 1, since in specifying the payoffs we can restrict to those that do not depend on unobserved fundamentals.

Appendix B. Proofs

Proof of Proposition 1. Fix a price $P > c$. For any strictly positive ε and ι , there is a realization of agent i 's signal that leads agent i to believe that Z exceeds $P/\psi(0) + \iota$ with probability at least $1 - \varepsilon$. For such a signal, agent i 's payoff from investing is at least

$$(1 - \varepsilon)(P/\psi(0) + \iota)\psi(0) - P = (1 - \varepsilon)\iota\psi(0) - \varepsilon P.$$

For any given $\iota > 0$, this is positive for ε small enough. Hence, for a given price $P > c$, there are realizations of the signal x_i for which investing is strictly dominant. Moreover, since the support of the noise term ε_i is unbounded, these realizations occur with positive probability. Hence, a monopoly supplier's expected profits from any input price $P > c$ are strictly positive. On the other hand, its expected profits from an input price $P \leq c$ are nonpositive. Finally, its profits from the price P are bounded above by $\psi(1)z[1 - G(z)]$ where $z = P/\psi(1)$. We now show that this bound goes to zero as $z \rightarrow \infty$. By L'Hôpital's rule,

$$\begin{aligned} & \lim_{z \rightarrow \infty} z[1 - G(z)] \\ &= \lim_{z \rightarrow \infty} \frac{1 - G(z)}{z^{-1}} = \lim_{z \rightarrow \infty} \frac{-G'(z)}{-z^{-2}} = \lim_{z \rightarrow \infty} [G'(z)z^2], \end{aligned}$$

which exists by assumption. Moreover, if $\lim_{z \rightarrow \infty} [G'(z)z^2] = a > 0$, then there must be a $z_0 < \infty$ such that for all $z > z_0$, $G'(z)z^2 \geq a/2$; but then by assumption,

$$\begin{aligned} \infty &> [1 - G(z_0)]E(Z|Z > z_0) = \int_{z=z_0}^{\infty} zG'(z) dz \\ &\geq \frac{a}{2} \int_{z=z_0}^{\infty} \frac{z}{z^2} dz = \frac{a}{2} \ln z \Big|_{z=z_0}^{\infty} = \infty, \end{aligned}$$

a contradiction. Accordingly, a monopoly supplier will choose a finite price $P > c$.

Now fix such a price. For any strictly positive ε and $\iota \in (0, P/\psi(1))$, there is a realization x_i of agent i 's signal that leads agent i to believe that Z is less than $P/\psi(1) - \iota$ with probability at least $1 - \varepsilon$. For such a signal, agent i 's payoff from investing is bounded above by

$$-\iota(1 - \varepsilon)\psi(1) + \varepsilon[E(Z|Z > P/\psi(1))\psi(1) - P].$$

Fixing ι , this upper bound is strictly negative for small enough ε . This shows that for any given price $P > c$, not investing is strictly dominant for low enough signals x_i . There are strategic complementarities as ψ is an increasing function. Finally, the payoff to investing is increasing in the signal x_i since $\psi(\ell) > 0$ for all ℓ . Hence, by Morris and Shin (2000a, Prop. 2.2), in the limit as $\tau \rightarrow 0$, each agent i invests if and only doing so is optimal under the belief that the proportion who will invest, ℓ , is uniformly distributed between zero and one. In addition, agents' signals all converge to $\ln Z$, so in the limit they invest if and only if $Z > P/\psi$.

Now consider the competitive case, where $P = c$. If $c > 0$ then by the preceding argument, the agents are playing a global game so they invest if and only if $Z > P/\Psi$. If $c = 0$ then investing is strictly dominant so agents always invest. Equivalently, they invest if and only if $Z > 0 = P/\Psi$ (which is always true). Hence, in both the competitive and monopoly cases, for any $c \geq 0$, the agents invest if and only if $Z > P/\Psi$ as claimed. \square

Proof of Claim 1. Let F be the distribution function of $B = \ln Z$, with associated hazard function $f(b) = \frac{F'(b)}{1-F(b)}$. By assumption, f is continuous and strictly increasing on $b \in \Re$ and satisfies $\lim_{b \rightarrow -\infty} f(b) = 0$ and $\lim_{b \rightarrow \infty} f(b) = \infty$. Since $G(z) = F(\ln Z)$, $g(z)(z - a) = f(\ln z) \frac{z-a}{z}$. If $a > 0$, then since both $f(\ln z)$ and $\frac{z-a}{z}$ are finite, continuous, and increasing functions of z on $[a, \infty)$, $g(z)(z - a)$ inherits these properties on $[a, \infty)$. Moreover,

$$\lim_{z \downarrow a} [g(z)(z - a)] = \lim_{z \downarrow a} \left[f(\ln z) \frac{z - a}{z} \right] = f(\ln a) * 0 = 0$$

and there is continuity in the limit (i.e., $g(z)(z - a) = 0$ when $z = a$); and

$$\begin{aligned} \lim_{z \rightarrow \infty} [g(z)(z - a)] &= \lim_{z \rightarrow \infty} \left[f(\ln z) \frac{z - a}{z} \right] \\ &= \lim_{z \rightarrow \infty} f(\ln z) * \lim_{z \rightarrow \infty} \frac{z - a}{z} = \infty. \end{aligned}$$

Finally, if $a = 0$, the above argument holds for $z \in (a, \infty)$. However, $g(0)$ is finite since $G'(0) < \infty$ and $1 - G(0) = 1$ (as G is continuous). Hence, $g(z)z$ is continuous on $[0, \infty)$. \square

Proof of Theorem 1. Since $Z \leq \kappa$ if and only if $u \leq \frac{\ln \kappa - \mu}{\sigma}$, the probability of a crisis is $\pi = H\left(\frac{\ln \kappa - \mu}{\sigma}\right)$. Inverting this gives

$$\kappa = \exp(\sigma H^{-1}(\pi) + \mu). \tag{5}$$

Social welfare equals $SW(c, \mu, \pi) = \int_{p=\pi}^1 (\psi(1) \exp(\sigma H^{-1}(p) + \mu) - c) dp$ by (4) and (5).

In the competitive case, $\kappa = c/\Psi$, so $\pi = H\left(\frac{\ln c - \ln \Psi - \mu}{\sigma}\right)$. Hence, $\frac{\partial \pi}{\partial c} > 0$, $\frac{\partial \pi}{\partial \mu} < 0$, and $\lim_{c \rightarrow 0} \pi = \lim_{\mu \rightarrow \infty} \pi = 0$. Since $\frac{\partial SW}{\partial c} < 0$, $\frac{\partial SW}{\partial \mu} > 0$, and $\frac{\partial SW}{\partial \pi} = c \left(1 - \frac{\psi(1)}{\psi}\right) \leq 0$ (which holds with equality only if $\Psi = \psi(1)$), social (and thus agent) welfare is decreasing in c and increasing in μ . Finally, since κ is fixed, the derivative of the crisis risk $H\left(\frac{\ln \kappa - \mu}{\sigma}\right)$ with respect to σ has the same sign as $\mu - \ln \kappa$, as claimed. This completes the proof for the competitive case (part 1).

The monopoly supplier's expected profits are $\Pi = \Psi \left(\kappa - \frac{c}{\Psi}\right) \left(1 - H\left(\frac{\ln \kappa - \mu}{\sigma}\right)\right)$. The derivative of Π with respect to κ is $\frac{1-H}{\sigma} \left(\sigma - \left(1 - \frac{c}{\kappa\Psi}\right) h\right)$ where H, H' , and h are evaluated at $\frac{\ln \kappa - \mu}{\sigma}$. Hence, the first order condition is $h\left(\frac{\ln \kappa - \mu}{\sigma}\right) \left(1 - \frac{c}{\kappa\Psi}\right) = \sigma$. The left hand side is continuous, strictly increasing in κ , equals zero at $\kappa = \frac{c}{\Psi}$, and exceeds σ in the limit as $\kappa \rightarrow \infty$ since the range of h includes an open neighborhood of σ . Hence, it has a unique solution $\kappa = \kappa^* \in \left(\frac{c}{\Psi}, \infty\right)$. Profits are increasing in κ for $\kappa < \kappa^*$ and decreasing for $\kappa > \kappa^*$, so κ^* is the global maximum.

Defining $x = \frac{\ln \kappa - \mu}{\sigma}$ and $\omega = e^{-\mu \frac{c}{\Psi}}$, we compute $\omega e^{-\sigma x} = e^{-\mu \frac{c}{\Psi}} e^{\mu - \ln \kappa} = \frac{c}{\kappa\Psi}$, so the first order condition can be written

$$1 = \frac{h(x)}{\sigma} (1 - \omega e^{-\sigma x}). \tag{6}$$

The right hand side of (6) is continuous in ω and x , strictly increasing in x , and strictly decreasing in ω . Hence, the solution x is

increasing in ω , and thus so is the probability $\pi = H(x)$ of a crisis, as claimed (part 2b). However, if ω increases, causing an increase in x and thus $h(x)$, then $\omega e^{-\sigma x}$ must also rise, since otherwise, the right hand side of (6) would exceed one. Thus, an increase in ω raises $\omega e^{-\sigma x} = \frac{c}{\kappa\Psi}$.

Now, the variable ω depends only on μ and c , since Ψ is fixed. Suppose the increase in ω is due to a change in just one of these parameters. If ω rises because μ rises, then $\frac{c}{\Psi}$ is unaffected, so κ and $P = \Psi\kappa$ must fall (part 2c). If ω rises because c rises, then since $x = \frac{\ln \kappa - \mu}{\sigma}$ rises while μ is unchanged, κ and thus P must rise (part 2c).

Now consider any sequence $(\omega_n)_{n=1}^\infty$ of positive numbers that converges monotonically to zero, and for each ω_n let x_n be the unique solution to (6) that corresponds to $\omega = \omega_n$. We claim that $\lim_{n \rightarrow \infty} x_n$ exists and equals $h^{-1}(\sigma)$ ²⁴: the solution to (6) when $\omega = 0$. If not, then there is an $\varepsilon > 0$ such that for every finite n^* , there is an $n > n^*$ such that $|x_n - h^{-1}(\sigma)| > \varepsilon$. Hence, there must exist an infinite set S of positive integers such that either (a) for every $n \in S$, $x_n - h^{-1}(\sigma) > \varepsilon$ or (b) for every $n \in S$, $x_n - h^{-1}(\sigma) < -\varepsilon$. Without loss of generality, assume (a) holds. Hence, since $x_n > h^{-1}(\sigma) + \varepsilon$ and since the right hand side of (6) is strictly increasing in x , it follows that $\sigma = h(x_n) (1 - \omega_n e^{-\sigma x_n}) > h(h^{-1}(\sigma) + \varepsilon) (1 - \omega_n e^{-\sigma(h^{-1}(\sigma) + \varepsilon)})$ for all $n \in S$. But S includes indices n for which ω_n is arbitrarily close to zero. Hence, it must be the case that $\sigma \geq h(h^{-1}(\sigma) + \varepsilon)$. But then $h^{-1}(\sigma) \geq h^{-1}(\sigma) + \varepsilon$, a contradiction. We have shown that as ω shrinks to zero, $x = \frac{\ln \kappa - \mu}{\sigma}$ falls, monotonically approaching $h^{-1}(\sigma)$, which is the value x takes when $\omega = 0$. Hence, the crisis probability $H(x)$ falls monotonically to the limit $H(h^{-1}(\sigma)) \in (0, 1)$ (part 2b) and $e^{-\mu} P = \Psi e^{\sigma x}$ falls monotonically (holding Ψ is fixed) to $\Psi e^{\sigma h^{-1}(\sigma)}$ (part 2c); and both quantities are continuous in the limit, as claimed.

The supplier's profits are increasing in μ and decreasing in c by the envelope theorem. By part 2b and the properties of $SW(c, \mu, \pi)$ shown above, the same is true of social welfare. Agent welfare equals $e^\mu \int_{p=\pi}^1 (\psi(1) e^{\sigma H^{-1}(p)} - \Psi e^{\sigma H^{-1}(\pi)}) dp$ by (3) and (5). This expression is increasing in μ and, since $\psi(1) \geq \Psi$, decreasing in π . Since π is decreasing in μ and increasing in c , it follows that agent welfare is increasing in μ and decreasing in c , as claimed (part 2a). \square

Proof of Theorem 2. On seeing the public signal $Y = y$, the supplier's posterior over the log fundamental $\ln Z$ is normal with mean $m(y) = \frac{s^2 \mu + \sigma^2 y}{s^2 + \sigma^2}$ and variance $\hat{\sigma}^2 = \frac{s^2 \sigma^2}{s^2 + \sigma^2}$ (e.g., DeGroot, 1975). That is, the supplier believes that $\ln Z = m(y) + \hat{\sigma}u$, where u is standard normal. As this functional form fits the assumptions of Theorem 1, we have the following corollary to that theorem. Let Φ be the standard normal distribution function and ζ be its (strictly increasing) hazard function. Let $P(y)$ and $\pi(y)$ be the supplier's optimal input price and crisis risk conditional on a realization y of the public signal Y . The corollary relies on the fact that the posterior mean $m(y)$ of the log fundamental is an increasing linear function of y .

Corollary 2. As either c shrinks to zero or the realized public signal y goes to infinity:

1. the conditional crisis probability $\pi(y)$ falls monotonically to its value when $c = 0$, which is $\Phi(\zeta^{-1}(\hat{\sigma}))$;
2. the conditional input price $P(y)$ rises (falls) if y rises (c falls), but in either case, the scaled input price $e^{-m(y)} P(y)$ converges to its value when $c = 0$, which is $\Psi e^{\hat{\sigma} \zeta^{-1}(\hat{\sigma})}$.

²⁴ As h is strictly increasing and continuous, it has an inverse h^{-1} that is also strictly increasing and continuous.

We now restrict to the case $c = 0$. By Corollary 2, the conditional crisis probability $\pi(y)$ is $\Phi(\zeta^{-1}(\hat{\sigma}))$, which is independent of the realized public signal y , and the conditional input price $P(y)$ is $\Psi e^{m(y) + \hat{\sigma}\zeta^{-1}(\hat{\sigma})}$. Hence, the agents' conditional log threshold $k = \ln \kappa$ equals $k(y) = \hat{\sigma}\zeta^{-1}(\hat{\sigma}) + m(y)$. Let $B = \ln Z$. The crisis risk conditional on $B = b$ is

$$\begin{aligned} \pi(b, s) &= \Pr_Y(k(Y) > B | B = b) \\ &= \Pr_Y(m(Y) + \hat{\sigma}\zeta^{-1}(\hat{\sigma}) > B | B = b) \\ &= \Pr_\epsilon\left(\frac{s^2\mu + \sigma^2b + \sigma^2\epsilon}{s^2 + \sigma^2} + \hat{\sigma}\zeta^{-1}(\hat{\sigma}) > b\right) \\ &= \Phi\left(\frac{s}{\hat{\sigma}}\zeta^{-1}(\hat{\sigma}) - \frac{s}{\sigma^2}(b - \mu)\right) \end{aligned}$$

which is continuous and strictly decreasing in the realization b . As for social welfare, by the law of iterated expectations,

$$\begin{aligned} \frac{SW}{\psi(1)} &= E_{B,Y}(e^B 1(B > k(Y))) \\ &= E_Y(E_B[e^B 1(B < k(Y)) | Y]) \\ &= E_Y(e^{m(Y)} E_B[e^{B-m(Y)} 1(B - m(Y) > \hat{\sigma}\zeta^{-1}(\hat{\sigma})) | Y]) \\ &= \int_{y=-\infty}^{\infty} e^{m(y)} \int_{b=-\infty}^{\infty} e^{b-m(y)} 1 \\ &\quad \times (b - m(y) > \hat{\sigma}\zeta^{-1}(\hat{\sigma})) d\Phi\left(\frac{b - m(y)}{\hat{\sigma}}\right) d\Phi \\ &\quad \times \left(\frac{y - \mu}{\sqrt{\sigma^2 + s^2}}\right) \\ &= \left[\int_{b'=\zeta^{-1}(\hat{\sigma})}^{\infty} e^{\hat{\sigma}b'} d\Phi(b')\right] E_Y(e^{m(Y)}) \\ &\quad \text{where } b' = \frac{b - m(y)}{\hat{\sigma}}. \end{aligned}$$

As Y is normal, $E_Y(e^{m(Y)}) = \exp(\mu + \frac{1}{2} \frac{\sigma^4}{\sigma^2 + s^2}) = \exp(\mu + \frac{1}{2}(\sigma^2 - \hat{\sigma}^2))$. Now, for any c_1, c_2 , $\int_{x=-\infty}^{\infty} e^{c_2x} d\Phi(x) = e^{c_2^2/2}$ and

$$\begin{aligned} \int_{x=c_1}^{\infty} e^{c_2x} d\Phi(x) &= \frac{1}{\sqrt{2\pi}} \int_{x=c_1}^{\infty} e^{c_2x - \frac{x^2}{2}} dx \\ &= \frac{1}{\sqrt{2\pi}} e^{c_2^2/2} \int_{x=c_1}^{\infty} e^{-\frac{(x-c_2)^2}{2}} dx \\ &= \frac{1}{\sqrt{2\pi}} e^{c_2^2/2} \int_{x'=c_1-c_2}^{\infty} e^{-\frac{(x')^2}{2}} db \\ &= e^{c_2^2/2} (1 - \Phi(c_1 - c_2)), \end{aligned}$$

so $\int_{x=c_1}^{\infty} e^{c_2x} d\Phi(x) = e^{c_2^2/2} (1 - \Phi(c_1 - c_2))$ and $\int_{x=-\infty}^{c_1} e^{c_2x} d\Phi(x) = e^{c_2^2/2} \Phi(c_1 - c_2)$, whence $\int_{b'=\zeta^{-1}(\hat{\sigma})}^{\infty} e^{\hat{\sigma}b'} d\Phi(b') = e^{\hat{\sigma}^2/2} (1 - \Phi(\zeta^{-1}(\hat{\sigma}) - \hat{\sigma}))$ and so

$$\frac{SW}{\psi(1)} = e^{\mu + \frac{1}{2}\sigma^2} (1 - \Phi(\zeta^{-1}(\hat{\sigma}) - \hat{\sigma})).$$

As $x - \zeta(x)$ is increasing in x (Sampford, 1953, Eq. (3), p. 130), $\zeta^{-1}(\hat{\sigma}) - \hat{\sigma}$ is increasing in $\hat{\sigma}$, so social welfare is decreasing in $\hat{\sigma}$ and thus in s as claimed (part 1). Since, by part 1 of Corollary 2, $\pi(y)$ is independent of y , the supplier's expected profit Π is

$$\begin{aligned} E_Y(\Psi e^{k(Y)} (1 - \Phi(\zeta^{-1}(\hat{\sigma}))) &= \Psi \exp\left(\mu + \frac{1}{2}(\sigma^2 - \hat{\sigma}^2) + \hat{\sigma}\zeta^{-1}(\hat{\sigma})\right) \\ &\quad \times (1 - \Phi(\zeta^{-1}(\hat{\sigma}))), \end{aligned}$$

as claimed. Since $\zeta(\zeta^{-1}(\hat{\sigma})) = \hat{\sigma}$, $\frac{d}{d\hat{\sigma}}\zeta^{-1}(\hat{\sigma}) = \frac{1}{\zeta'(\zeta^{-1}(\hat{\sigma}))}$, so $\frac{\partial \Pi}{\partial \hat{\sigma}} = (\zeta^{-1}(\hat{\sigma}) - \hat{\sigma}) \Pi$, which is negative since $\zeta(x) > x$ for all $x \in \mathfrak{R}$ (Sampford, 1953, Eq. (3), p. 130). This proves part 1. As for part 2, Corollary 2 implies that conditional on the realized fundamental y , the crisis risk is $\Phi(\zeta^{-1}(\hat{\sigma}))$ and the input price P is $\Psi e^{\hat{\sigma}\zeta^{-1}(\hat{\sigma}) + m(y)}$. As the variance s^2 of the public signal shrinks to zero, $\hat{\sigma}^2 \rightarrow 0$ as well, the crisis risk and input price converge to $\Phi(-\infty) = 0$ and $\Psi e^{m(y)} = \Psi Z$, respectively. This proves part 2. \square

Proof of Claim 2. If $X_{it}^1 - f(i, X_{it}^{-1})$ and $X_{i,t+1}^1 - f(i, X_{i,t+1}^{-1})$ have opposite signs and each has an absolute value of at least δ , then the absolute difference between $X_{it}^1 - f(i, X_{it}^{-1})$ and $X_{i,t+1}^1 - f(i, X_{i,t+1}^{-1})$ is at least 2δ . However, since f is Lipschitz with constant λ , this absolute difference is also at most $(1 + \lambda) \max_k |X_{i,t+1}^k - X_{it}^k|$. Hence, $\max_k |X_{i,t+1}^k - X_{it}^k| > \frac{2\delta}{1+\lambda}$. By Large Jump Rarity, for all $\epsilon > 0$, there is a $\delta' < \infty$ and a time t^* such that at all times $t > t^*$,

$$\int_{i=0}^1 P_{t,t+1}^i \left(\max_k |X_{i,t+1}^k - X_{it}^k| > \delta'\right) di < \epsilon.$$

The claim follows by setting $\delta = \delta'(1 + \lambda)/2$. \square

Proof of Lemma 1. For any $c_2 > 0$, let $t_1(c_2)$ equal the time t^* that corresponds to setting $\epsilon = \epsilon_1/2$, $c = c_2 + 2b_1$, and $f(i, X_{it}^{-1}) = c(i, X_{it}^{-1}) - c_0(i) - b_1$ in Asymptotic Unpredictability. For all $t > t_1(c_2)$,

$$\begin{aligned} \int_{i=0}^1 P_t^i(X_{it}^1 \in [c(i, X_{it}^{-1}) - c_2 - b_1, c(i, X_{it}^{-1}) + b_1]) di \\ = \int_{i=0}^1 P_t^i(X_{it}^1 \in [f(i, X_{it}^{-1}), f(i, X_{it}^{-1}) + c]) di < \epsilon_1/2 \end{aligned}$$

where the last line follows from Asymptotic Unpredictability. The result follows by taking c_2 large enough that it exceeds $c_0(i)$ for at least a proportion $1 - \epsilon_1/2$ of the players. \square

Proof of Lemma 2. For any $\eta \in \mathfrak{R}$, let $\Sigma_i^U(\eta)$ be the set of signals $X_{it} \in \mathfrak{R}^k$ satisfying $X_{it}^1 \geq c(i, X_{it}^{-1}) + \eta$. Let $\Sigma_i^D(\eta)$ be the set of signals $X_{it} \in \mathfrak{R}^k$ satisfying $X_{it}^1 \leq c(i, X_{it}^{-1}) + \eta$. Then the event that player i starts at least κ away from her nondominance region in period t and then moves into or over her nondominance region in period $t + 1$ is just the event either (a) $X_{it} \in \Sigma_i^U(\kappa)$ and $X_{i,t+1} \in \Sigma_i^D(0)$, or (b) $X_{it} \in \Sigma_i^D(-c_0(i) - \kappa)$ and $X_{i,t+1} \in \Sigma_i^U(-c_0(i))$. By Large Reversal Rarity, there is a κ and a t^* such that for $t > t^*$,

$$\int_{i=0}^1 P_{t,t+1}^i(X_{it} \in \Sigma_i^U(\kappa), X_{i,t+1} \in \Sigma_i^D(0)) di < \epsilon_2/2.$$

(To see this, set $f(i, X_{it}^{-1}) = c(i, X_{it}^{-1}) + \kappa/2$.) Also by Large Reversal Rarity there is a κ' and $t^{*'}$ such that for $t > t^{*'}$,

$$\begin{aligned} \int_{i=0}^1 P_{t,t+1}^i[X_{it} \in \Sigma_i^D(-c_0(i) - \kappa), X_{i,t+1} \in \Sigma_i^U(-c_0(i))] di \\ < \epsilon_2/2. \end{aligned}$$

(To see this, set $f(i, X_{it}^{-1}) = c(i, X_{it}^{-1}) - c_0(i) - \kappa'/2$.) The result follows by setting $b_2 = \max\{\kappa, \kappa'\}$ and $t_2 = \max\{t^*, t^{*'}\}$. \square

Proof of Theorem 3. Let ϵ be the probability given in Vanishing Fluctuations, and $\epsilon_1 = \epsilon_2 = \epsilon/2$. Let t_2 and b_2 satisfy the conditions of Lemma 2 given ϵ_2 . Let $b_1 = b_2$ and let t_1 satisfy the condition of Lemma 1 given ϵ_1 and b_1 . Finally, let t^* for Vanishing Fluctuations equal $\max\{t_1, t_2\}$. Now suppose $t > t^*$. This implies $t > t_1$, so the expected proportion of players who are at least b_1 away from their nondominance regions is at least $1 - \epsilon_1$ by Lemma 1. Moreover, since $t > t_2$ and $b_2 = b_1$, the

expected proportion of players who are at least b_1 away from their nondominance regions in period t and who enter or skip over their nondominance regions in period $t + 1$ is at least $1 - \varepsilon_2$ by Lemma 2. Therefore, the expected proportion of players for whom both of these events happen at least $1 - \varepsilon_1 - \varepsilon_2 = 1 - \varepsilon$. In this case, X_{it}^1 must be on the same side of the interval $[c(i, X_v^{-1}) - c_0(i), c(i, X_v^{-1})]$ for both $v = t$ and $v = t + 1$: player i has the same, strictly dominant action in periods t and $t + 1$. Consequently, for any $t > t^*$, the expected proportion of players who do not have the same strictly dominant action in periods t and $t + 1$ is less than ε . \square

Proof of Theorem 4. First, suppose that Asymptotic Unpredictability fails while Large Reversal Rarity holds. We will produce payoffs such that Dominance Regions holds and Vanishing Fluctuations fails. Since Asymptotic Unpredictability fails, there is a positive $c < \infty$, a probability $\varepsilon > 0$, and a function $f \in F$ such that for all $t^* < \infty$ there is a time $t > t^*$ such that the ex ante expected proportion of players i who get signals X_{it}^1 in the interval $[f(i, X_{it}^{-1}), f(i, X_{it}^{-1}) + c]$ is at least ε . Suppose that each player's payoff function is as follows. If $X_{it}^1 < f(i, X_{it}^{-1})$, i gets payoff 1 from $a_t^i = 0$ and zero from all other actions. If $X_{it}^1 > f(i, X_{it}^{-1}) + c$, i gets payoff 1 from $a_t^i = 1$ and zero from all other actions. If $X_{it}^1 \in [f(i, X_{it}^{-1}), f(i, X_{it}^{-1}) + c]$, then all actions give the same payoff (say, zero). (Note that payoffs do not depend directly on the fundamental vector $B_{t \cdot}$.) By construction, if X_{it}^1 lies in the interval $[f(i, X_{it}^{-1}), f(i, X_{it}^{-1}) + c]$ then i has no dominant action and, moreover, her behavior is indeterminate. Since, by assumption, the proportion of such players is expected, ex ante, to exceed ε , Vanishing Fluctuations does not hold. In addition, we cannot rule out fluctuating behavior among these players since they are indifferent among all actions. However, Dominance Regions holds with $\bar{a}_i = 1, \underline{a}_i = 0, c_0(i) = c, c(i, X_{it}^{-1}) = f(i, X_{it}^{-1}) + c$, and λ equal to the Lipschitz constant of f .

Now suppose that Large Reversal Rarity fails. Then (setting $\delta = 0$) there is a Lipschitz constant $\lambda > 0$, probability $\varepsilon > 0$, and function f in F^λ , such that at for all $t^* < \infty$ there is a time $t > t^*$ such that the ex ante expected proportion of players for whom $X_{it}^1 - f(i, X_{it}^{-1}) > 0$ and $X_{i,t+1}^1 - f(i, X_{i,t+1}^{-1}) < 0$ is at least ε . Suppose that each player i gets a payoff of 1 from action $a_t^i = 1$ and zero from all other actions if $X_{it}^1 - f(i, X_{it}^{-1}) > 0$, and a payoff of 1 from action $a_t^i = 0$ and zero from all other actions if $X_{i,t+1}^1 - f(i, X_{i,t+1}^{-1}) < 0$. (As before, payoffs do not depend directly on the fundamental vector.) In this case, Vanishing Fluctuations fails and, moreover, behavior actually fluctuates: there is an infinite sequence of periods t in which the proportion of players who change their actions is at least ε . However, Dominance Regions holds, setting $\bar{a}_i = 1, \underline{a}_i = 0, c_0(i) = 0, c(i, X_{it}^{-1}) = f(i, X_{it}^{-1})$, and λ equal to the Lipschitz constant of f .

Dominance Regions can fail in four mutually exclusive ways. In this part we assume private values.

1. There may be a positive-measure set \bar{S} of players such that for each i in this set, there is a vector y_i^{-1} such that i lacks an upper or lower dominance region in X_{it}^1 when $X_{it}^{-1} = y_i^{-1}$. That is, for each player $i \in \bar{S}$ and for any integer $n = 0, 1, \dots$ there are realizations y_{in}^1 and z_{in}^1 of X_{it}^1 , both greater than n or both less than $-n$, such that the same action is not strictly dominant when X_{it} equals (y_{in}^1, y_i^{-1}) as when it equals (z_{in}^1, y_i^{-1}) . W.l.o.g. assume that both realizations exceed n . Clearly, we can choose the sequences so that $y_{in}^1 + 2 < z_{in}^1 + 1 < y_{i,n+1}^1$ for all $n \geq 0$. Now consider the following state distribution. X_{it}^{-1} always equals y_i^{-1} . Moreover, X_{i0}^1 equals y_{i0}^1 . For $t = 0, 1, \dots$, if $X_{it}^1 = y_{in}^1$, then $X_{i,t+1}^1$ equals y_{in}^1 and z_{in}^1 with equal

probabilities. Likewise, if $X_{it}^1 = z_{in}^1$, then $X_{i,t+1}^1$ equals z_{in}^1 and $y_{i,n+1}^1$ with equal probabilities. This process clearly satisfies Asymptotic Unpredictability and Large Reversal Rarity, yet in any two adjacent periods, at least one half of the players in \bar{S} do not have the same strictly dominant strategy, so Vanishing Fluctuations fails.

2. It may be that there is a full measure set of players who have upper and lower dominance regions in X_{it}^1 for all X_{it}^{-1} , but there exists a positive-measure set \bar{S} of players for whom the actions played in the upper (lower) dominance regions depend on X_{it}^{-1} : for each player $i \in \bar{S}$, there are two realizations y_i^{-1} and z_i^{-1} of X_{it}^{-1} such that for any $n < \infty$ there are realizations y_{in}^1 and z_{in}^1 , both greater than n or both less than $-n$, such that the same action is not strictly dominant when X_i equals (y_{in}^1, y_i^{-1}) as when it equals (z_{in}^1, z_i^{-1}) . W.l.o.g. assume that both realizations exceed n . Clearly, we can choose the sequence $(y_{in}^1)_{n=0}^\infty$ so that $y_{i,n+1}^1 > y_{in}^1 + 1$ for all $n = 0, 1, \dots$, and we can choose the sequence $(z_{in}^1)_{n=0}^\infty$ in the same way. Now consider the following state distribution. X_{it}^{-1} equals y_i^{-1} when t is odd and z_i^{-1} when it is even. Moreover, X_{i0}^1 equals y_{i0}^1 . For $t \geq 0$, if $X_{it}^1 = y_{in}^1$, then $X_{i,t+1}^1$ equals z_{in}^1 and $y_{i,n+1}^1$ with equal probabilities. Likewise, if $X_{it}^1 = z_{in}^1$, then $X_{i,t+1}^1$ equals y_{in}^1 and $y_{i,n+1}^1$ with equal probabilities. This process clearly satisfies Asymptotic Unpredictability and Large Reversal Rarity, yet in any two adjacent periods, at least one half of the players in \bar{S} do not have the same strictly dominant strategy, so Vanishing Fluctuations fails.
3. There may be a full-measure set S of players who have upper and lower dominance regions in X_{it}^1 for all X_{it}^{-1} , such that for each player in S the same action is strictly dominant in all the upper (lower) dominance regions, but there exists a positive-measure subset \bar{S} of S for whom the size of the nondominance region is not a bounded function of the signal-vector X_{it}^{-1} . Then for each player $i \in \bar{S}$, and for each $\delta < \infty$, there is a signal-vector $w_{i\delta}^{-1} \in \mathfrak{R}^{K-1}$ such that when $X_{it}^{-1} = w_{i\delta}^{-1}$, player i 's nondominance region $[y_{i\delta}^1, z_{i\delta}^1] \subset \mathfrak{R}$ satisfies $z_{i\delta}^1 - y_{i\delta}^1 > \delta$. In this case, let $(\alpha_{in})_{n=0}^\infty$ be a sequence of i.i.d. $U[0, 1]$ random variables. For any player $i \in \bar{S}$, define the sequence of K -vectors $V_{in} = (y_{in}^1 + \alpha_{in}(z_{in}^1 - y_{in}^1), w_{in}^{-1})$ for $n = 0, 1, \dots$. Let $(\theta_t)_{t=0}^\infty$ be a random sequence defined as follows: $\theta_0 = 1$ and for $t > 0$, θ_t equals θ_{t-1} with probability $1 - 1/t$ and $\theta_{t-1} + 1$ with probability $1/t$. Consider a state distribution defined by $X_{it} = V_{i\theta_t}$ for all t . The players in \bar{S} are never in their dominance regions, so Vanishing Fluctuations fails. However, Asymptotic Unpredictability and Large Reversal Rarity clearly hold.
4. Finally, there may be a full measure set of players that has upper and lower dominance regions in X_{it}^1 for all X_{it}^{-1} , such that the size of each such player's nondominance region is bounded in X_{it}^{-1} , if there exists a positive measure subset \bar{S} of S such that for each player $i \in \bar{S}$, the upper boundary $c(i, X_{it}^{-1})$ of the nondominance region is not Lipschitz in X_{it}^{-1} , and cannot be made Lipschitz by choosing a larger interval size $c_0(i)$. We will use the following claim. Recall that $\|\cdot, \cdot\|$ denotes Euclidean distance.

Claim 3. Under the conditions assumed in this part, for any $i \in \bar{S}$ and $\delta, \delta' < \infty$, there are $y, z \in \mathfrak{R}^{K-1}$ satisfying $\|y, z\| \leq \delta$ such that $|c(i, y) - c(i, z)| > \delta'$.

Proof. Assume not. Then there exist $i \in \bar{S}$ and $\delta, \delta' < \infty$ such that for all $y, z \in \mathfrak{R}^{K-1}$ satisfying $\|y, z\| \leq \delta$, we have $|c(i, y) - c(i, z)| \leq \delta'$. Let L be the lattice in \mathfrak{R}^{K-1} consisting of vectors each component of which is an integer multiple of $\frac{\delta}{K-1}$. On elements of L , $c(i, \cdot)$ is Lipschitz-continuous with constant δ'/δ , since for any $y, z \in L$, $|c(i, y) - c(i, z)| \leq \sum_{k=1}^{K-1} \frac{|y_k - z_k|}{\delta} \delta'$.

By McShane (1934), there exists a Lipschitz continuous function $\tilde{c} : \mathfrak{R}^{K-1} \rightarrow \mathfrak{R}$ with Lipschitz constant δ'/δ , such that \tilde{c} equals $c(i, \cdot)$ on L . For any $y \in \mathfrak{R}^{K-1}$, let $\lfloor y \rfloor$ denote the greatest element of L that does not exceed y in any component. For any $y \in \mathfrak{R}^{K-1}$, $\|y, \lfloor y \rfloor\| \leq \delta$, so

$$\begin{aligned} |c(i, y) - \tilde{c}(y)| \\ \leq |c(i, y) - c(i, \lfloor y \rfloor)| + |c(i, \lfloor y \rfloor) - \tilde{c}(y)| \leq 2\delta'. \end{aligned}$$

Hence, for all $y \in \mathfrak{R}^{K-1}$, $c(i, y) \in [\tilde{c}(y) - 2\delta', \tilde{c}(y) + 2\delta']$. Thus, i 's nondominance region is contained in $[\hat{c}(y) - \hat{c}_0, \hat{c}(y)]$ where $\hat{c}(y) = \tilde{c}(y) + 2\delta'$ and $\hat{c}_0 = c_0(i) + 4\delta'$. As $\hat{c}(y)$ is Lipschitz, this is a contradiction. \square

Now fix $i \in \bar{S}$. By Claim 3, for each $n = 0, 1, \dots$, there exist $y_n, z_n \in \mathfrak{R}^{K-1}$ satisfying $\|y_n, z_n\| \leq 1$ such that $|c(i, y_n) - c(i, z_n)| > n$. Let $(\alpha_n)_{n=0}^\infty$ be a sequence of i.i.d. $U[0, 1]$ variables and for each n , define $\beta_n = \alpha_n c(i, z_n) + (1 - \alpha_n) c(i, y_n)$. Let $(\gamma_t)_{t=0}^\infty$ be a sequence of i.i.d. variables, each of which equals 0 or 1 with equal probabilities. Let $(\theta_t)_{t=0}^\infty$ be a sequence of variables, such that $\theta_0 = 1$ and for $t > 0$, θ_t equals θ_{t-1} with probability $1 - 1/t$ and $\theta_{t-1} + 1$ with probability $1/t$. Define X_{it}^1 to equal β_{θ_t} and let X_{it}^{-1} equal z_{θ_t} if $\gamma_t = 0$ and y_{θ_t} if $\gamma_t = 1$. Since X_{it}^1 is uniform on an interval of length at least θ_t , which goes to infinity with probability one, and the relative location of X_{it}^1 in this interval is independent of X_{it}^{-1} , Asymptotic Unpredictability is satisfied. Since $\|z_{\theta_t}, y_{\theta_t}\| \leq 1$, with probability $1 - 1/t$, $\|X_{i,t+1}, X_{it}\| \leq 1$, so Large Jump Rarity holds. This implies Large Reversal Rarity by Claim 2. Finally, the probability is at least $1 - 1/t$ that player i is not in the same dominance region in periods t and $t + 1$, so Vanishing Fluctuations fails. \square

References

- Angeletos, George-Marios, Hellwig, Christian, Pavan, Alessandro, 2006. Signalling in a global game: coordination and policy traps. *Journal of Political Economy* 114, 452–484.
- Angeletos, George-Marios, Hellwig, Christian, Pavan, Alessandro, 2007. Dynamic global games of regime change: learning, multiplicity, and the timing of attacks. *Econometrica* 75, 711–756.
- Angeletos, George-Marios, Werning, Ivan, 2006. Crises and prices: information aggregation, multiplicity, and volatility. *American Economic Review* 96, 1720–1736.
- Argenziano, Rossella, 2008. Differentiated networks: equilibrium and efficiency. *The RAND Journal of Economics* 39, 747–769.
- Aumann, Robert J., 1974. Subjectivity and correlation in randomized strategies. *Journal of Mathematical Economics* 1, 67–96.
- Barlow, Richard, Proschan, Frank, 1975. *Statistical Theory of Reliability and Life Testing*. Holt, Rinehart & Winston, New York.
- Blackwell, David, 1953. Equivalent comparisons of experiments. *Annals of Mathematical Statistics* 24, 265–272.
- Burdzy, Krzysztof, Frankel, David M., 2005. Shocks and business cycles. *Advances in Theoretical Economics* 5 (1), Paper No. 2.
- Carlsson, Hans, van Damme, Eric, 1993. Global games and equilibrium selection. *Econometrica* 61, 989–1018.
- Cass, David, Shell, Karl, 1983. Do sunspots matter? *Journal of Political Economy* 91, 193–227.
- Chamley, Christophe, 1999. Coordinating regime switches. *Quarterly Journal of Economics* 114, 869–905.
- Cournot, Antoine Augustin, 1897. *Researches into the Mathematical Principles of the Theory of Wealth*. MacMillan, New York.
- DeGroot, Morris H., 1975. *Probability and Statistics*. Addison-Wesley, Reading, Mass.
- Farrell, Joseph, Katz, Michael, 1998. The effects of antitrust and intellectual property law on compatibility and innovation. *Antitrust Bulletin* 609–650. Fall/Winter.
- Frankel, David M., Morris, Stephen, Pauzner, Ady, 2003. Equilibrium selection in global games with strategic complementarities. *Journal of Economic Theory* 108, 1–44.
- Frankel, David M., Pauzner, Ady, 2000. Resolving indeterminacy in dynamic settings: the role of shocks. *Quarterly Journal of Economics* 115, 283–304.
- Frankel, David M., Pauzner, Ady, 2002. Expectations and the timing of neighborhood change. *Journal of Urban Economics* 51, 295–314.
- Goldstein, Itay, Pauzner, Ady, 2004. Contagion of self-fulfilling financial crises due to diversification of investment portfolios. *Journal of Economic Theory* 119, 151–183.
- Goldstein, Itay, Pauzner, Ady, 2005. Demand-deposit contracts and the probability of bank runs. *Journal of Finance* 60, 1293–1327.
- Granger, Clive, 1981. Some properties of time series data and their use in econometric model specification. *Journal of Econometrics* 16, 121–130.
- Guimaraes, Bernardo, 2006. Dynamics of currency crises with asset market frictions. *Journal of International Economics* 68, 141–158.
- Jin, Yu., 2011. Bank monitoring and liquidity in the secondary market for loans. Mimeo.
- McShane, E.J., 1934. Extension of range of functions. *Bulletin of the American Mathematical Society* 40, 837–842.
- Morris, Stephen, Shin, Hyun Song, 1998. Unique equilibrium in a model of self-fulfilling currency attacks. *American Economic Review* 88, 587–597.
- Morris, Stephen, Shin, Hyun Song, 2000a. Global games: theory and applications. In: Dewatripont, M., Hansen, L., Turnoksky, S. (Eds.), *Advances in Economic Theory and Econometrics: Proceedings of the Eighth World Congress of the Econometric Society*. Cambridge University Press.
- Morris, Stephen, Shin, Hyun Song, 2000b. Rethinking multiple equilibria in macroeconomic modelling. *NBER Macroeconomics Annual* 15, 139–161.
- Morris, Stephen, Shin, Hyun Song, 2004. Liquidity black holes. *Review of Finance* 8, 1–18.
- Nadarajah, Saralees, 2005. A generalized normal distribution. *Journal of Applied Statistics* 32, 685–694.
- Oyama, Daisuke, 2004. Booms and slumps in a game of sequential investment with the changing fundamentals. *The Japanese Economic Review* 55, 311–320.
- Sampford, M.R., 1953. Some inequalities on Mill's ratio and related functions. *Annals of Mathematical Statistics* 24, 130–132.
- Toxvaerd, Flavio, 2008. Strategic merger waves: a theory of musical chairs. *Journal of Economic Theory* 140, 1–26.